# Writtens Workshop Fall 2016 : Linear Algebra

Aman Gupta

## 1 Key Topics :

- **Basics :** Solutions to systems Linear Equations, Rank and Nullity, Matrix as Linear Maps, Change of basis, Determinant, Trace, Characteristic polynomials, Matrix Operators
- **Matrix Decompositions :** Eigenvalue, Polar, LU, Cholesky , SVD decompositions, Schur's Decomposition, Jordan Form and Nilpotency
- **Inner Product spaces :** Inner Products, Norms, Orthogonal Projections, Linear functionals and Adjoints, Inequalities, Gram-Schmidt orthogonalizations
- **Special Operators :** Spectral Theorem, Theory of Symmetric/Hermitian matrices, Normal Matrices, Positive Definite matrices, Orthogonal/Unitary Matrices, Affine Subspaces, Rotation and Householder reflection, Quadratic Forms
- **Misc. :** Power Method, Rayleigh Iteration, Courant min-max principle, Gershgorin circle theorem, Matrix exponential, Coordinate transformation, Matrix Equations, Stochastic Matrices, Norm-minimization problems

## 2 Applications of Linear Algebra:

Linear Algebra finds application in virtually all branches of modern mathematics :

- **Numerical Methods** : Circulant & Toeplitz Matrices from Advection-Diffusion Equation Upwind Schemes, FFT Solvers, Tridiagonal Matrices from Cubic Splines, Chebyshev basis
- **Quadratic Forms and Convex Optimization** : Symmetric solvers like Jacobi, Gauss-Seidel solvers, Method of Conjugate Gradient, Stationary point analysis in Multivariable Calculus
- **Machine Learning and Data Science stuff** : Least Squares, Gauss-Newton Method, Vandermonde Matrices
- **Economics** : Linear Optimization Theory and Simplex Method
- **Evaluating Integrals** : Using quadratic forms and coordinate transformations to compute cryptic multivariate integrals
- **Mechanics and Robotics** : Rotation, Reflector Matrices and Orthogonal transformations
- **Markov Chains** : Stochastic Matrices, Poisson Process
- **Linear ODE Theory** : Matrix Exponential and Jordan Normal Form, Phase-Plane and fixed point analysis of IVPs, Variation or Parameters, Wronskians
- **Perturbation Theory** : Approximate solution to perturbed Eigenvalue problems
- **Image Processing** : Heavy use of DFT and SVD Decompositions
- **Graph Theory** : Representing Graphs as Matrices, Finding lengths of Cycles, connectedness of graphs
- **Recurrence Relations** : Fibonacci Series

## 3 Aim and Structure of this workshop

- Approach : Problem Solving oriented. Theory covered in the courses.
- Week 1 : Theory specific problems 1
- Week 2 : Theory specific problems 2
- Week 3 : Problems inspired from applications

## 4 Discussion

- Students' Background
- Any problem with the format?
- Any topics in particular that we should discuss?
- Show workshop outline
- Not discussing some basic topics like Linear Equations, Characteristic polynomial, Interpolation, LU and Gaussian Elimination, Power Method, SVD, QR, Gram-Schmidt, Differentiation operator. Does that work?

## 5 Additional Resources

1. Study Circles : facilitates exchange of ideas and removing doubts
2. Courant Math Wiki. Past year problems and solutions.
3. *Berkeley Problems in Mathematics* : a bit inclined to the theoretical side but a great problem supplement.
4. Additional problems if required.

## 6 Topics not covered :

- Linear Equation, Characteristic Polynomials
- Power Method, Eigenvalue Decomposition, SVD, LU, QR
- Least Squares, Gram-Schmidt Orthogonalization