Diagnosing the impact of numerics on stratospheric transport in GCMs

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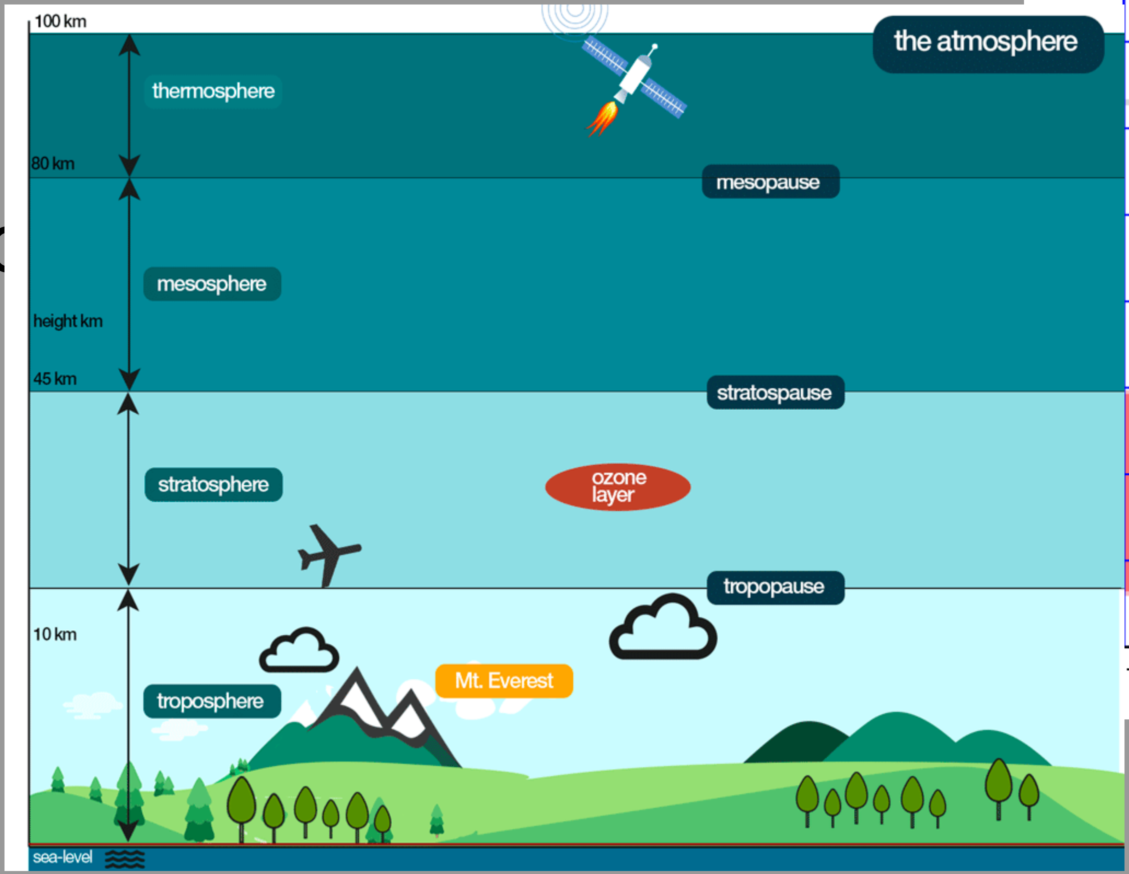
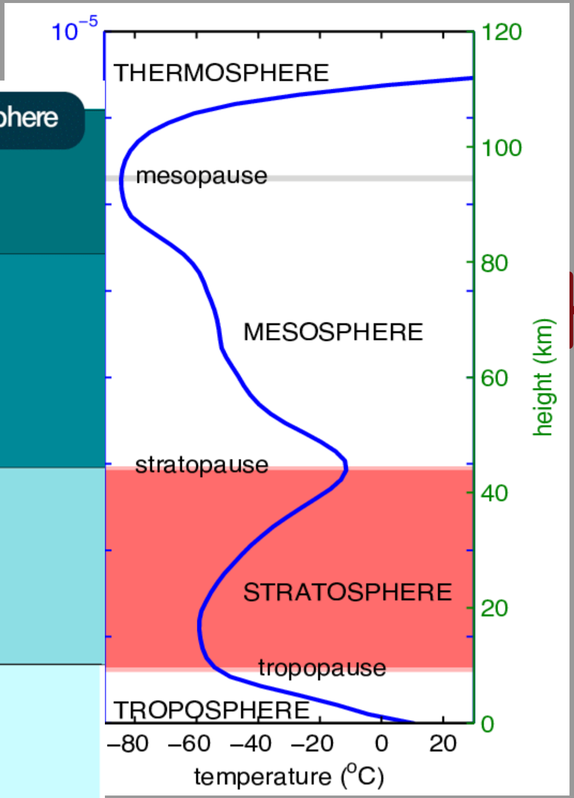
Diagnosing the impact of numerics on STRATOSPHERIC transport in GCMs

Diagn HERIC

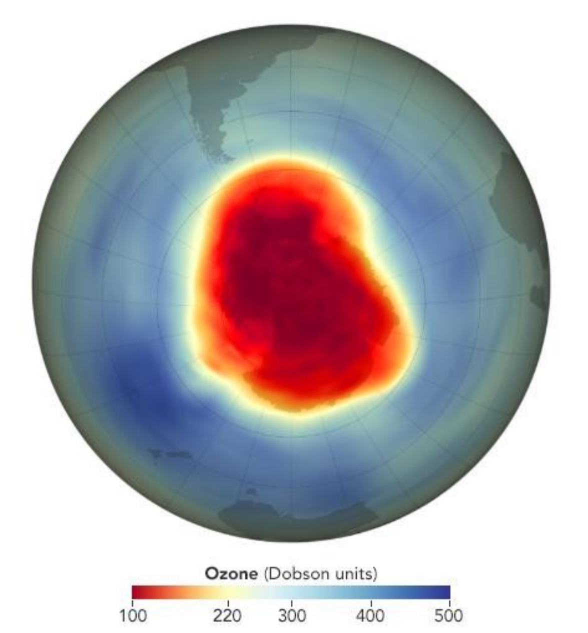


100 km
the atmosphere
thermosphere
80 km
mesopause
mesosphere
height km
45 km
stratospause
stratosphere
ozone layer
tropopause
10 km
Mt. Everest
troposphere
sea-level



10^{-5}
THERMOSPHERE
mesopause
MESOSPHERE
stratopause
STRATOSPHERE
tropopause
TROPOSPHERE
120
100
80
60
40
20
0
height (km)
−80 −60 −40 −20 0 20
temperature (°C)

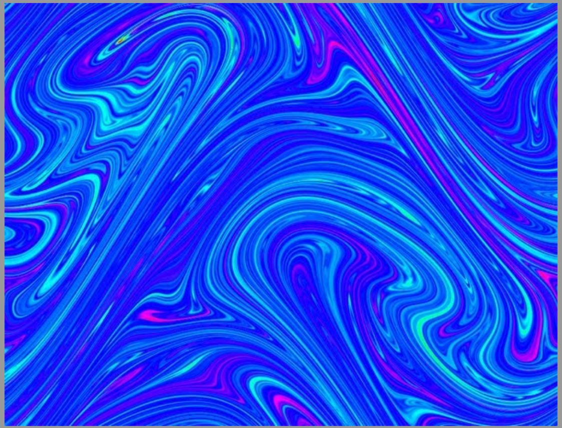
Diagnosing the impact of numerics on stratospheric TRANSPORT in GCMs



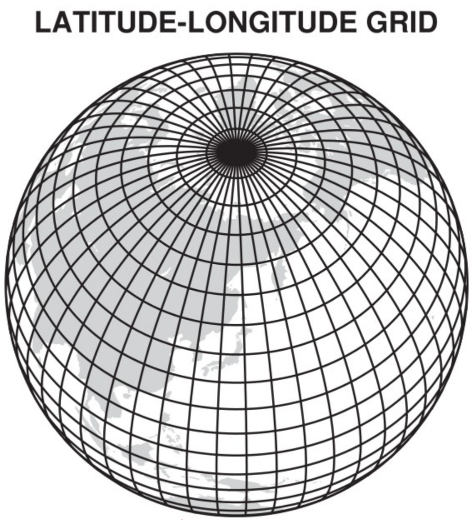
D he impact of numerics on TRANSPORT in GCMs

$$c_t + \vec{u}.\nabla c = \kappa \nabla^2 c$$

$$c_t = \kappa \nabla^2 c$$



Diagnosing the impact of NUMERICS on stratospheric transport in GCMs



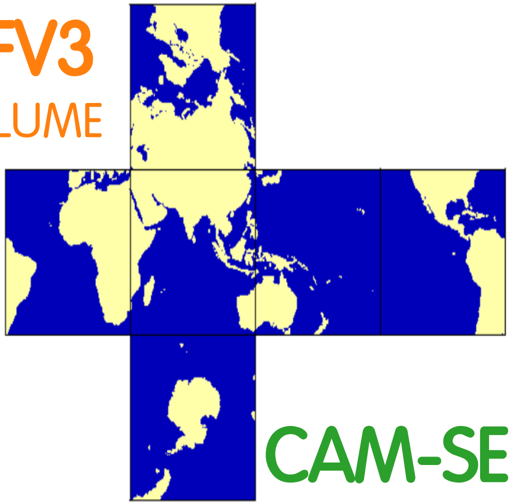
CAM-FV

CAM FINITE VOLUME

e impact of NUMERICS
transport in GCMs

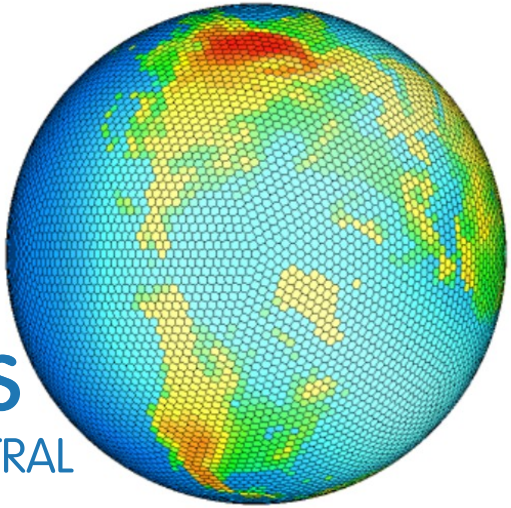
GFDL-FV3

CUBED SPHERE FINITE VOLUME



CAM-SE

CAM SPECTRAL ELEMENT



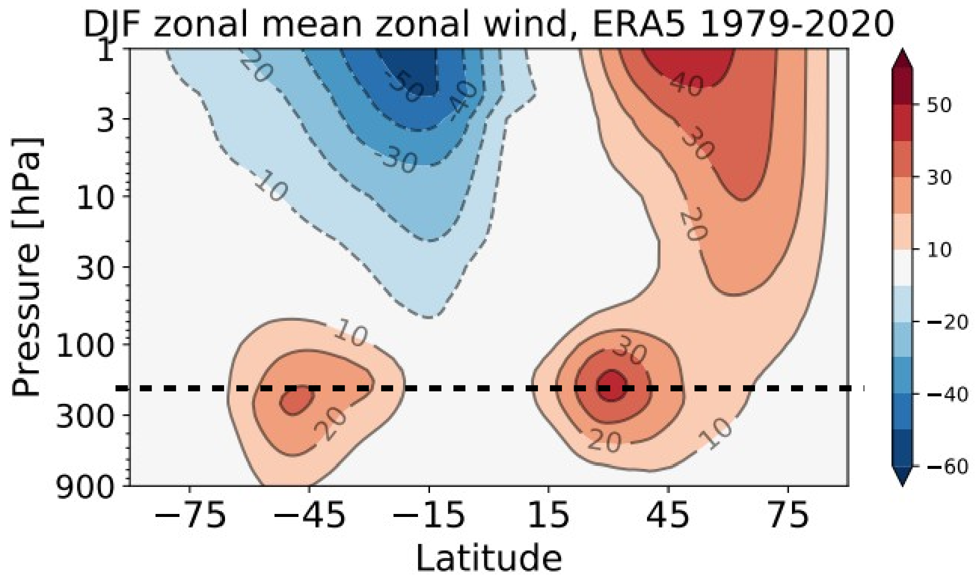
GFDL-PS

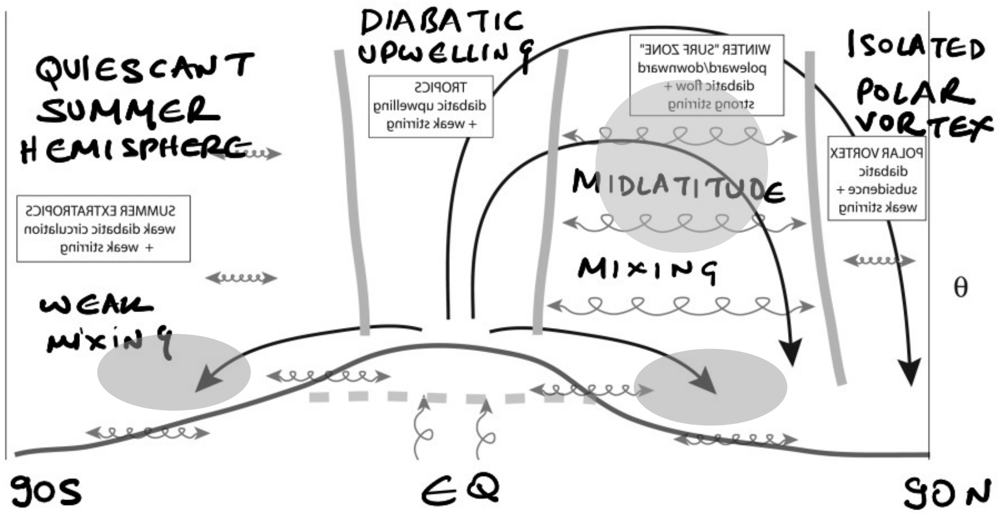
PSEUDOSPECTRAL

Outline

- **Introduction and Motivation :** overview of the stratospheric circulation, transport and idealized models
- **The Leaky Pipe :** A theoretical framework to study transport in the stratosphere
- **Results :** Using the leaky pipe to study the impact of numerics and dynamics in determining stratospheric transport

DJF climatology in the stratosphere

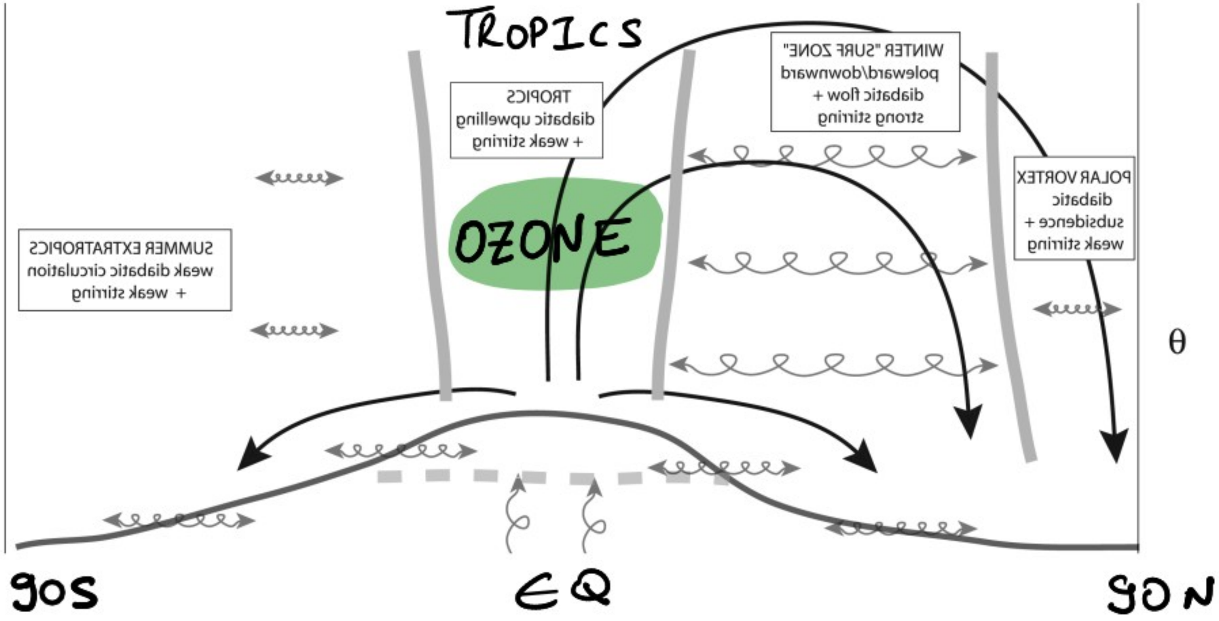




Climatological winds : Wintertime polar vortex and tropical and summer hemisphere easterlies

Tropics dominated by **diabatic upwelling**, midlatitudes by **isentropic mixing**, isolated **polar night jet** over the winter poles. Quiescent summer hemisphere.

Poleward Transport by Stratospheric Circulation



Most ozone is produced in the tropics, yet at the onset of spring, the ozone concentration is the highest near the winter poles.

First evidence of the tropics-to-pole circulation we now refer to as the Brewer-Dobson Circulation (BDC)

Quantifying transport : Age of Air (AoA)

- **Age of air** of an air-mass quantifies the time elapsed last surface contact. (Hall and Plumb 1994, Waugh and Hall '02)
- **A measure of transport timescales** in the atmosphere. Estimates residence times of tracer gases like CFCs, SF_6, CH_4
- An idealized tracer with a source in time. Independent of chemistry/parameterizations, only depends on the tracer advection suite of a model.

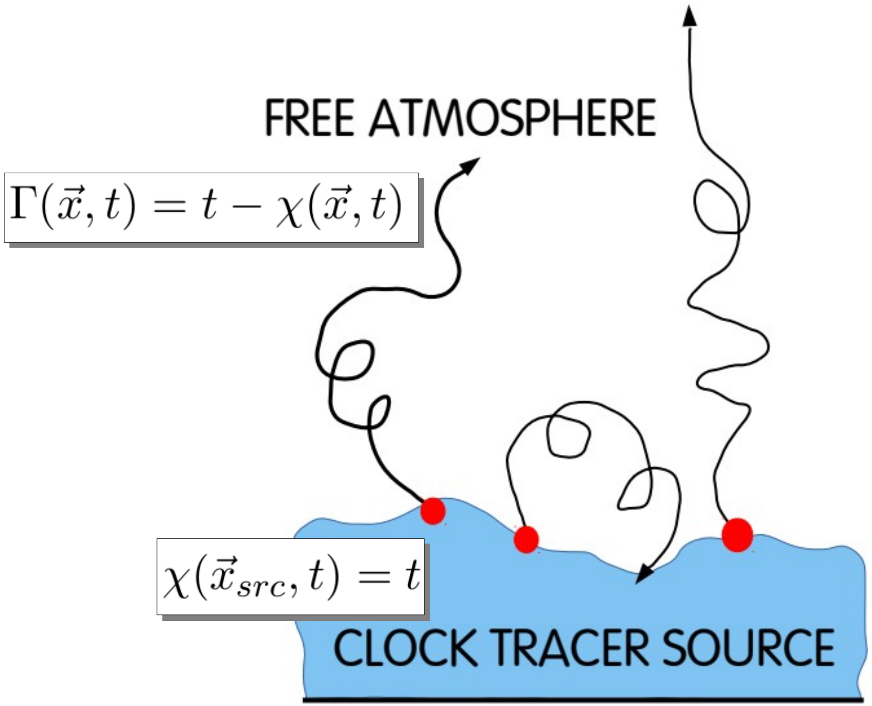
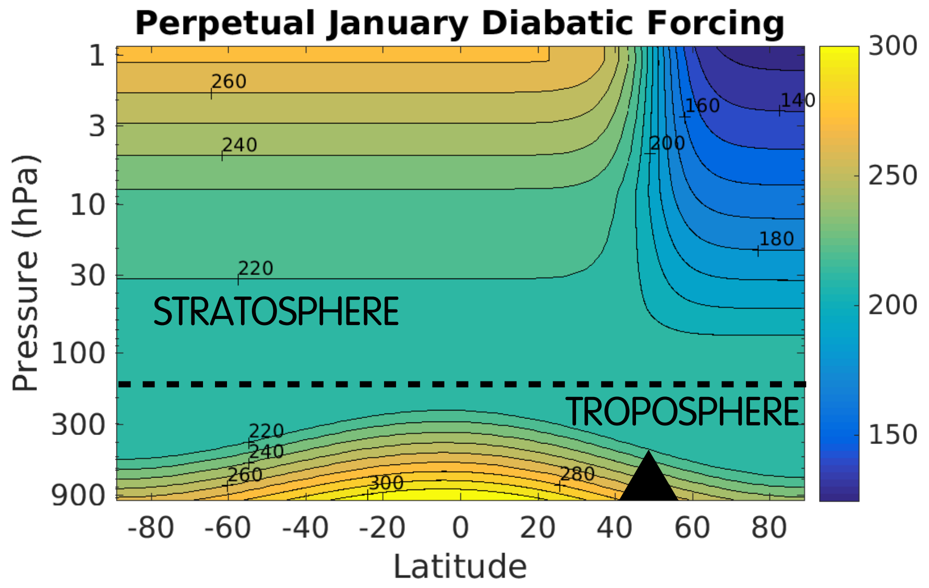


Fig : Computing age in models using clock tracer

Model stratosphere

- Models – useful tools to study the stratosphere!
- Using **idealized models** provide a way to focus on large-scale dynamics and transport coupling. **No parameterizations**, less uncertainty.
- **Newtonian relaxation** to idealized diabatic forcings to get a fairly realistic representation of tropospheric and stratospheric circulation
- Using a **clock tracer** at the surface to compute the age-of-air in idealized models

$$\frac{\partial T}{\partial t} = \cdots - k_T(\phi,\sigma)\left[T - T_{eq}(\phi,p)\right]$$



This setup is called a Held-Suarez and Polvani-Kushner (HSPK) setup

Studying transport in an idealized framework

- Use the setup to force the 2 dynamical cores (fluid equation solver) :

  - Identical **idealized diabatic forcing** (HSPK); wave-2 topography at 45N
  - No seasonal cycle : DJF climatology
  - Integrated for 10,000 days
  - **No parameterizations**
  - Linearly increasing clock tracer

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CAM-SE

CAM SPECTRAL ELEMENT

Uses spectral finite element in the horizontal and finite volume in the vertical

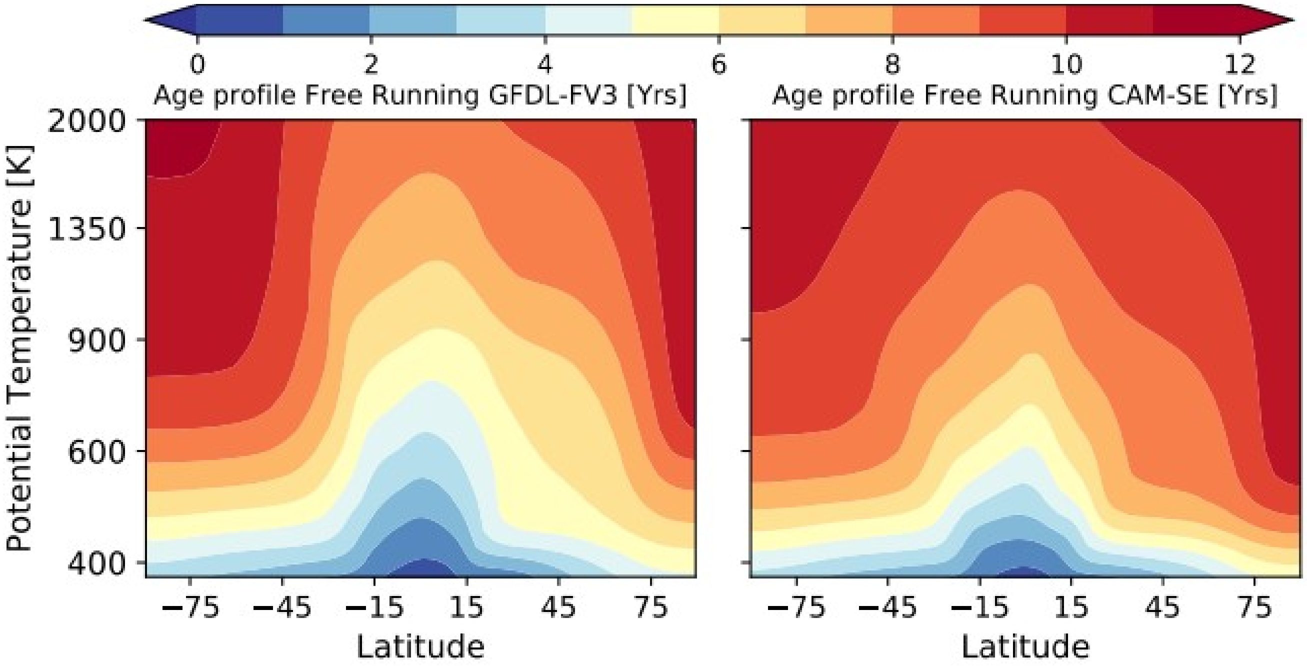
GFDL-FV3

CUBED SPHERE FINITE VOLUME

Uses finite volume conservative formulation in the horizontal and the vertical

- Comparable horizontal and vertical resolution (1deg x 1deg, 80 vertical levels)

Strikingly different age-of-air in modern dynamical cores



FV3

FINITE VOLUME

SE

SPECTRAL ELEMENT

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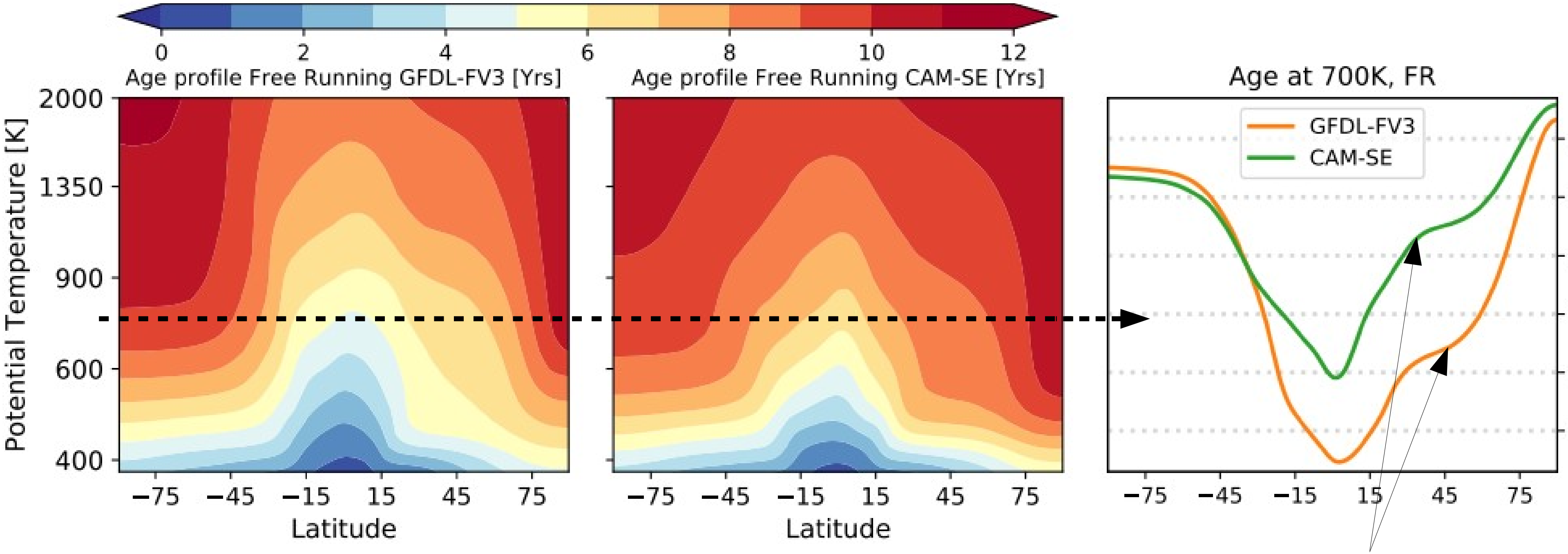


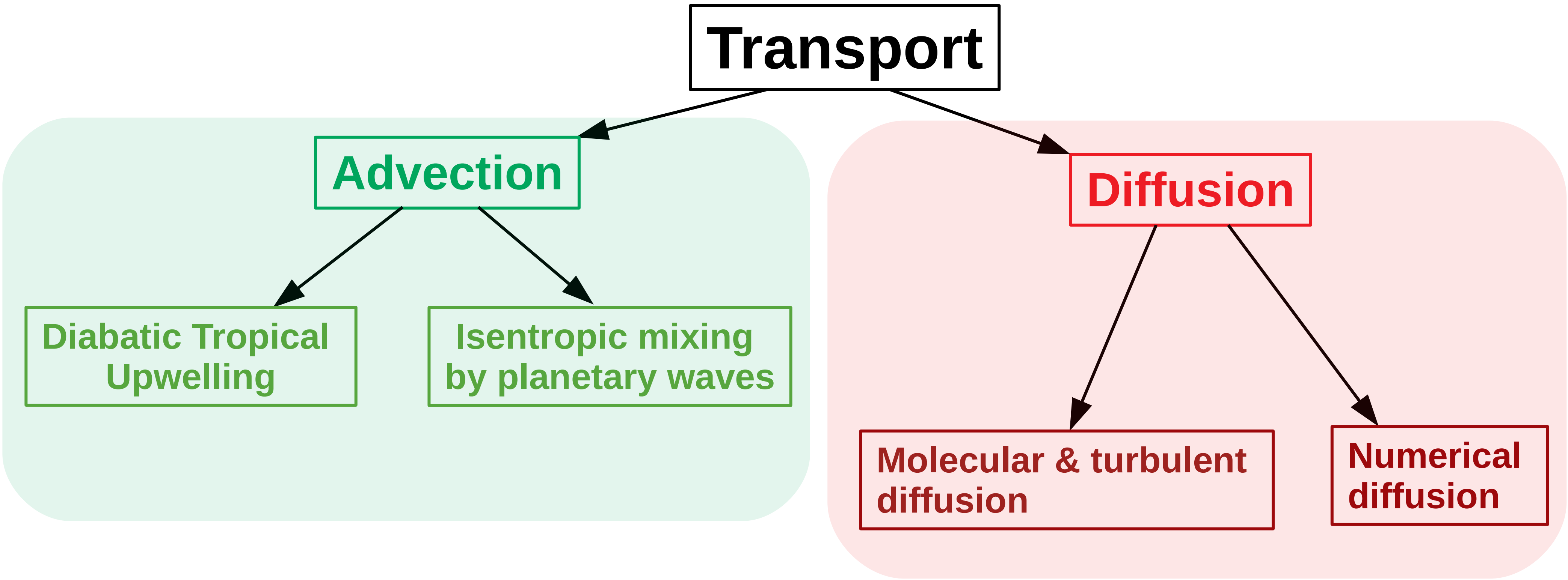
FV3
FINITE VOLUME

SE
SPECTRAL ELEMENT

Age of air in SE is 40% older than FV3

The 2 cores develop strikingly different age-of-air profiles in the stratosphere. Why?

What factors affect stratospheric transport?



The Problem

Question : When we force two different “modern” dynamical cores with identical diabatic forcings, why do they develop such different transport?

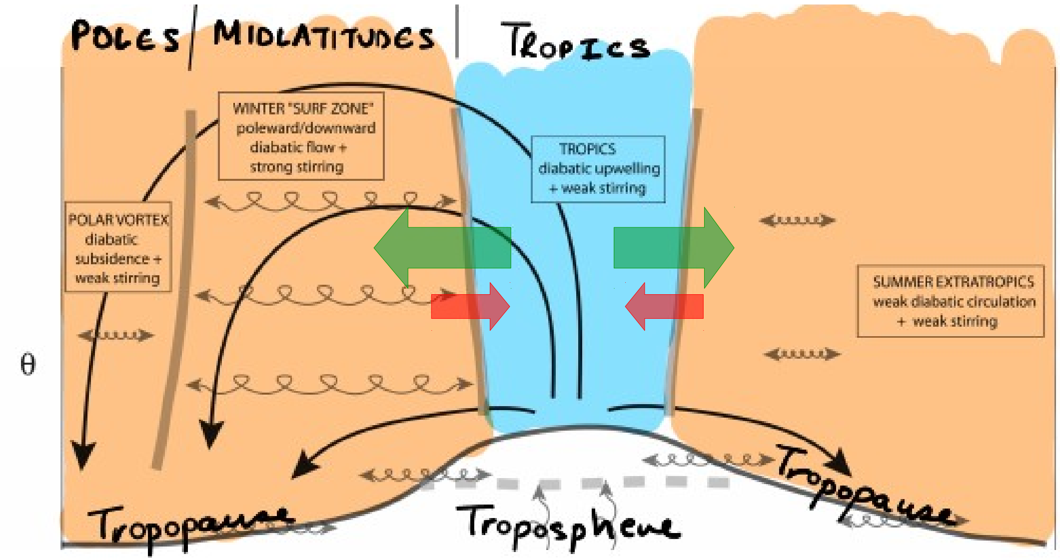
Question : What is causing these differences?

- Differences in numerical diffusion in tracer advection schemes?
- Differences in isentropic mixing in midlatitude stratosphere?
- Differences in diabatic circulation among the models?

Question : Can we assess the extent to which each factor affects differences in transport?

The Theoretical Leaky Pipe Model of Stratospheric Transport

Zonally averaged stratospheric circulation



- The **TLP model** (Neu and Plumb '99) integrates and divides the stratosphere into 2 regions of upwelling (u) and downwelling (d). **Hydrostatic. Isothermal. Steady state.**

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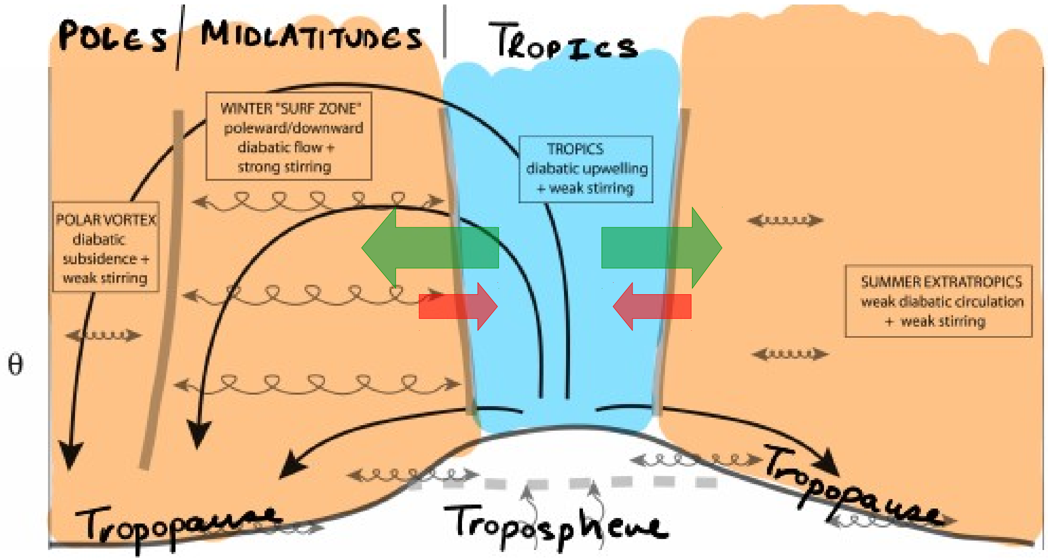
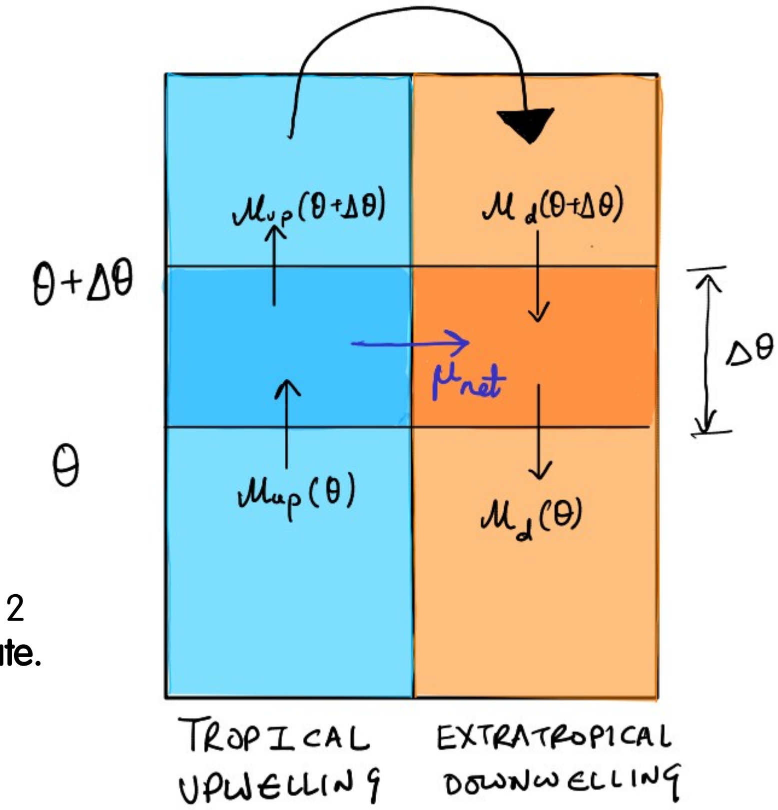
Zonally averaged stratospheric circulation

horizontally integrating

Simplified schematic and fluxes in leaky pipe





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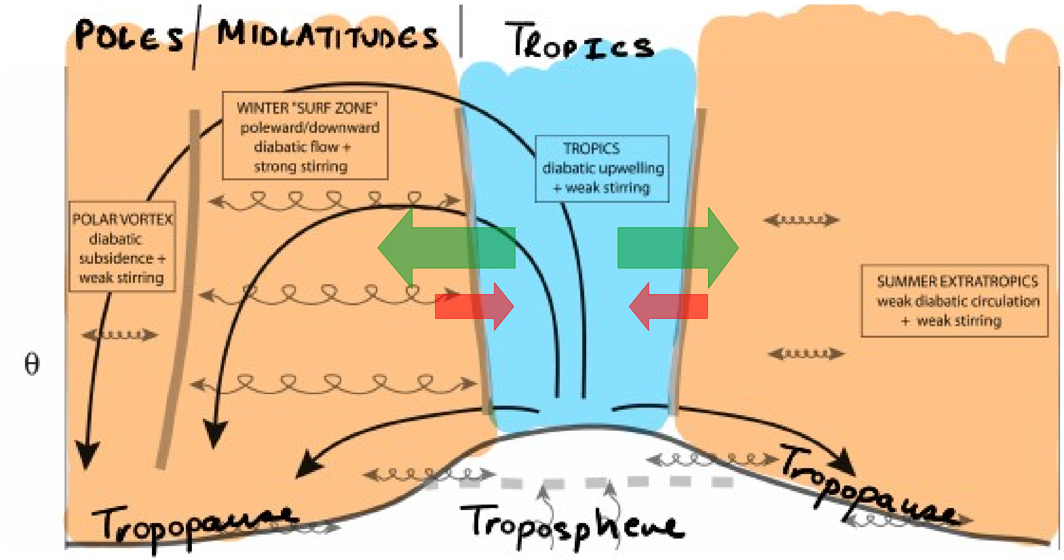

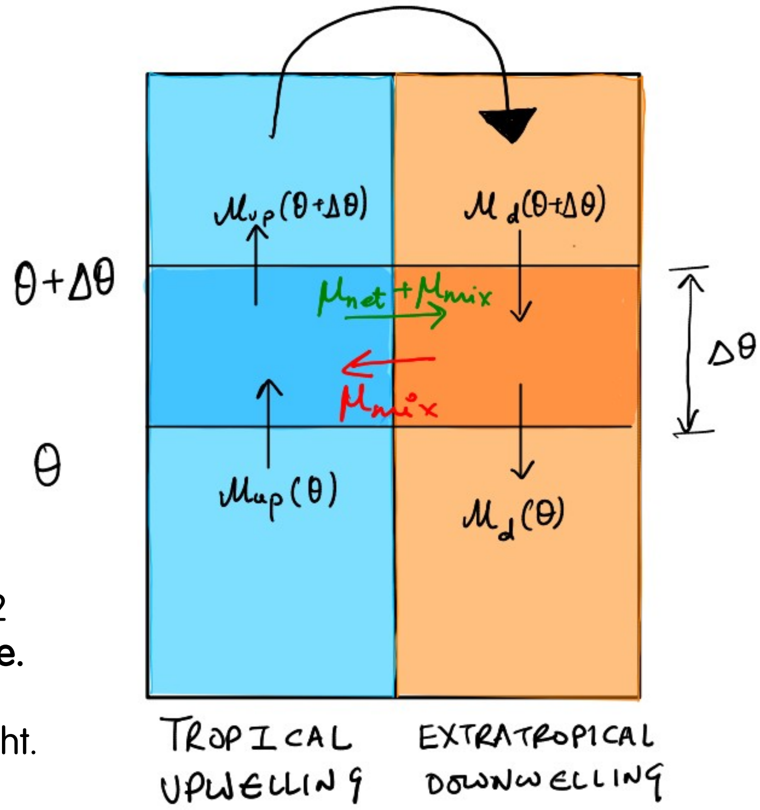

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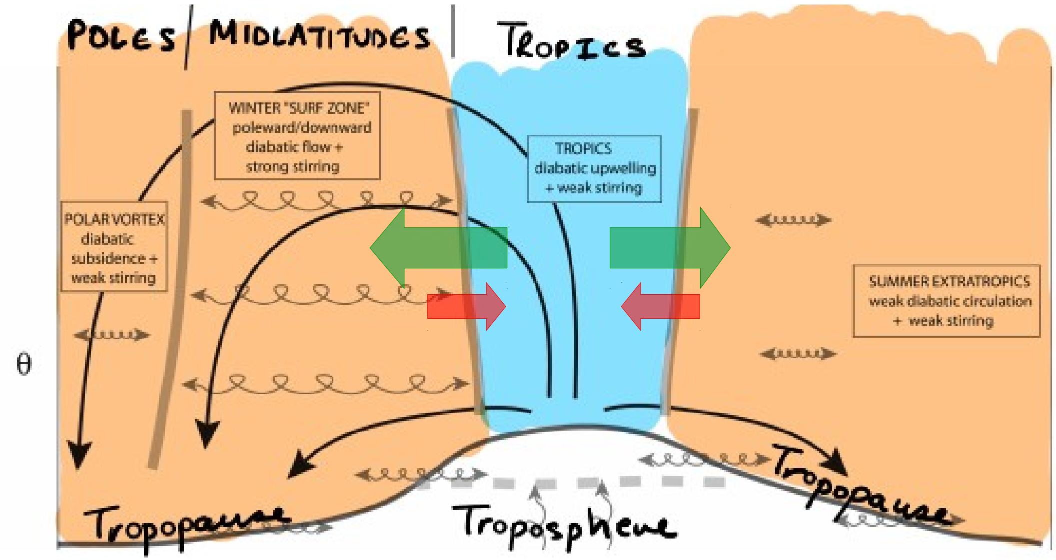
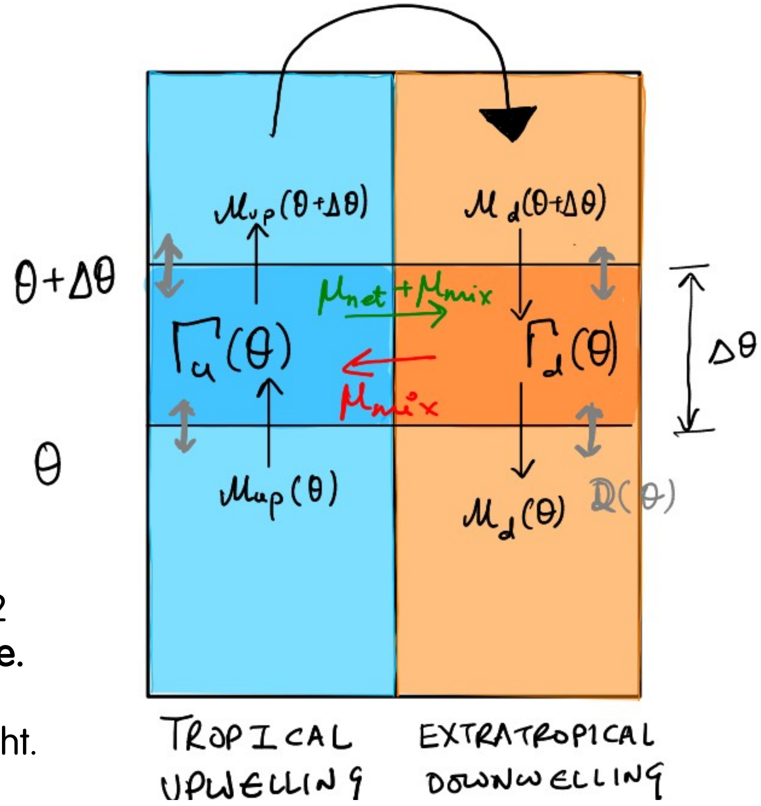
The Theoretical Leaky Pipe Model of Stratospheric Transport

Zonally averaged stratospheric circulation

horizontally integrating

Simplified schematic and fluxes in leaky pipe





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We allow vertical variations in all Leaky Pipe parameters

- The original leaky pipe formulation uses constant vertical velocity and mixing efficiency. While it makes the problem analytically solvable, it prevents a direct connection between models and theory.

- **We allow vertical variations of all the leaky pipe parameters** and reformulate it in isentropic coordinates, for a more accurate model-to-theory connection.

- These equations are numerically integrated with ascent rate, mixing efficiency and mass distribution determined using model data

Theoretical leaky pipe formulation

$$-K_M \frac{\partial^2 \Gamma_M}{\partial Z^2} + \left(\frac{K_M}{H} - \alpha W_T\right)\frac{\partial \Gamma_M}{\partial Z} - (1+\epsilon)\lambda(\Gamma_T - \Gamma_M) = 1$$

$$-K_T \frac{\partial^2 \Gamma_T}{\partial Z^2} + \left(\frac{K_T}{H} + W_T\right)\frac{\partial \Gamma_T}{\partial Z} + \frac{\epsilon\lambda}{\alpha}(\Gamma_T - \Gamma_M) = 1$$

diffusion

ascent rate

mixing efficiency

Stratospheric mass distribution

$W_T = 0.3 mm\ s^{-1}$
$\alpha = 1/2$
$\epsilon = 1$

$W_T \rightarrow W_T(Z)$
$\alpha \rightarrow \alpha(Z)$
$\epsilon \rightarrow \epsilon(Z)$

Computed from full model data

$$-K_M \frac{\partial^2 \Gamma_M}{\partial Z^2} + \left(\frac{K_M}{H} - \alpha(Z)W_T(Z)\right)\frac{\partial \Gamma_M}{\partial Z} - (1+\epsilon(Z))\lambda(Z)(\Gamma_T - \Gamma_M) = 1$$

$$-K_T \frac{\partial^2 \Gamma_T}{\partial Z^2} + \left(\frac{K_T}{H} + W_T(Z)\right)\frac{\partial \Gamma_T}{\partial Z} + \frac{\epsilon(Z)\lambda(Z)}{\alpha(Z)}(\Gamma_T - \Gamma_M) = 1$$

Creating parallelism between full model transport and 1D theoretical Leaky Pipe model

LEAKY PIPE	CLIMATE MODELS
0.3×10^{-6}km s^{-1}	Vertical velocity W_T
0.5	Mass distribution (α)
$\frac{-\alpha W_T}{H}$	Entrainment Ratio (λ)
constant $\in [0,1]$	Mixing (ϵ)
$\Gamma_T(Z)$	Upwelling Age (Γ_u)
$\Gamma_M(Z)$	Downwelling Age (Γ_d)

Creating parallelism between full model transport and 1D theoretical Leaky Pipe model

LEAKY PIPE		CLIMATE MODELS
$0.3 \times 10^{-6}\text{km s}^{-1}$	Vertical velocity W_T	$\dot{\theta}\left\|\frac{d\theta}{dp}\right\| = \frac{\int_{up} \rho_\theta \dot{\theta} dA}{\int_{up} \rho_\theta dA}\left\|\frac{d\theta}{dp}\right\|$
0.5	Mass distribution (α)	$\frac{\rho_{\theta,u}}{\rho_{\theta,u}+\rho_{\theta,d}}$
$\frac{-\alpha W_T}{H}$	Entrainment Ratio (λ)	$-\partial_\theta(\mathcal{M}_{up})/\rho_{\theta,d}$
constant $\in [0,1]$	Mixing (ϵ)	μ_{mix}/μ_{net}
$\Gamma_T(Z)$	Upwelling Age (Γ_u)	$\Gamma_u(\theta) = \frac{\int_u \rho_\theta \dot{\theta} \Gamma \, dA}{\int_u \rho_\theta \dot{\theta} \, dA}$
$\Gamma_M(Z)$	Downwelling Age (Γ_d)	$\Gamma_d(\theta) = \frac{\int_d \rho_\theta \dot{\theta} \Gamma \, dA}{\int_d \rho_\theta \dot{\theta} \, dA}$

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The missing piece : estimating the mixing flux

1. Age of air is a tracer with a source in time :

$$\frac{\partial \Gamma}{\partial t} + \frac{1}{\rho_\theta}\nabla \cdot \mathcal{F} = 1$$

$\Gamma \equiv \Gamma(x, y, \theta, t)$: age of air

$\rho_\theta = \frac{1}{g}|dp/d\theta|$: isentropic density

$\mathcal{F} = \rho_\theta \dot{\theta}\Gamma - \kappa\nabla(\rho_\theta \Gamma)$: advective-diffusive flux of age

2. In the no-diffusion limit, the vertical gradient of these quantities allow quantifying the mixing fluxes across the subtropical barrier (Linz et al. *in prep*)

Vertical gradient :

$$\frac{\partial \Gamma_u}{\partial \theta} = \frac{\sigma}{\mathcal{M}} + \frac{\mu_{mix}\Delta\Gamma}{\mathcal{M}}$$

aging by advection — aging by mixing

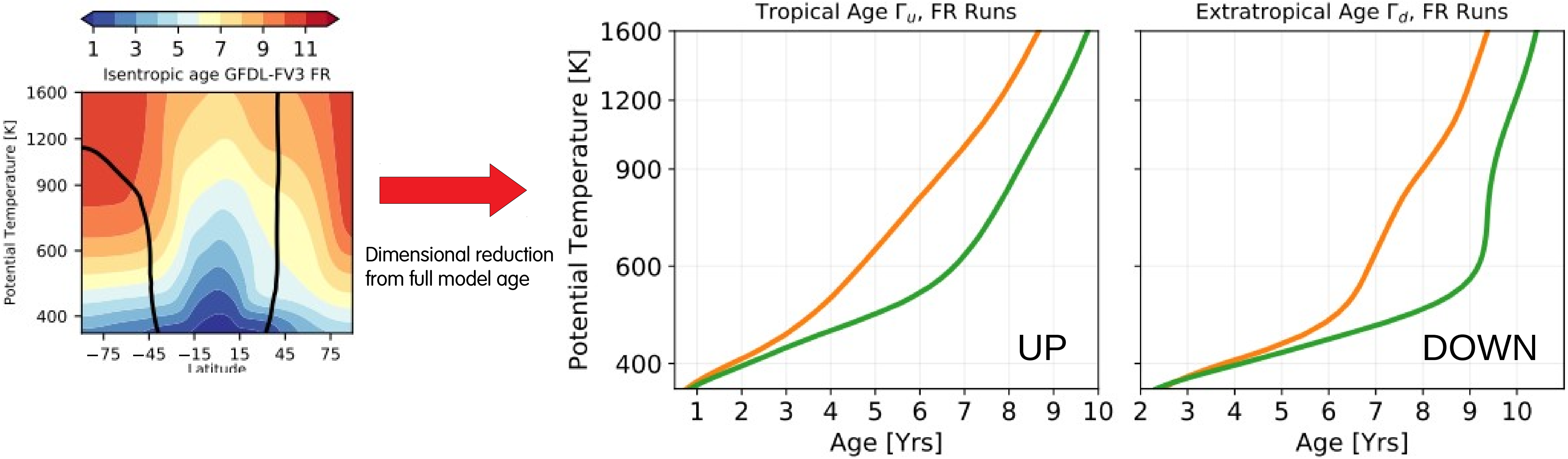
$\mathcal{M}(\theta)$: diabatic mass flux

$\sigma(\theta)$: horizontally avgd density

$\mu_{mix}(\theta)$: ET $\rightarrow$ T mixing flux

We compute the weighted age Γ_u and Γ_d and μ_{mix} from the 2 climate models, impose them in the leaky pipe "emulator" and incrementally change parameters to see their effect on the age.

Comparing the model age and the leaky pipe "fit"



A good fit is obtained between integrated age from models and the age from vertically varying leaky pipe formulation in both the regions.

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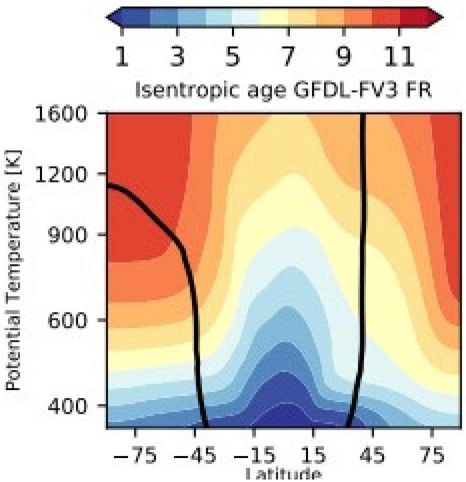
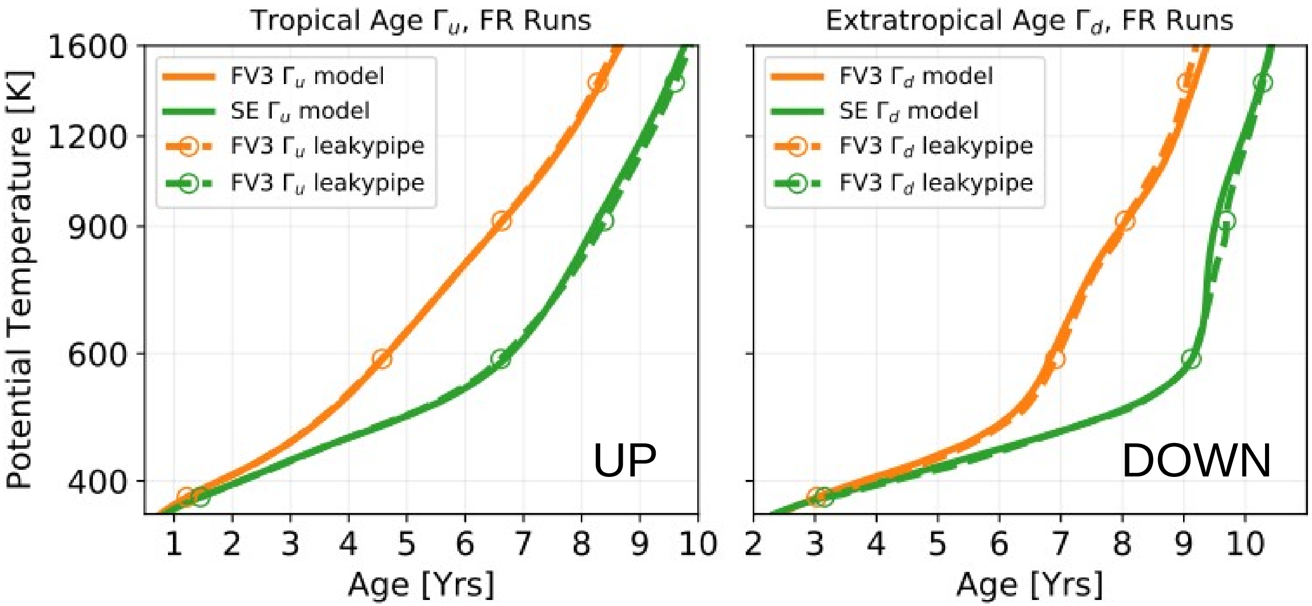


Dimensional reduction from full model age

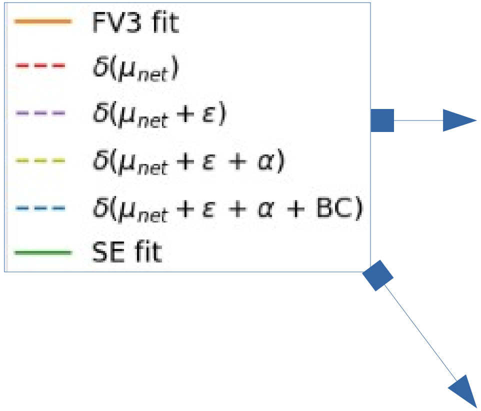


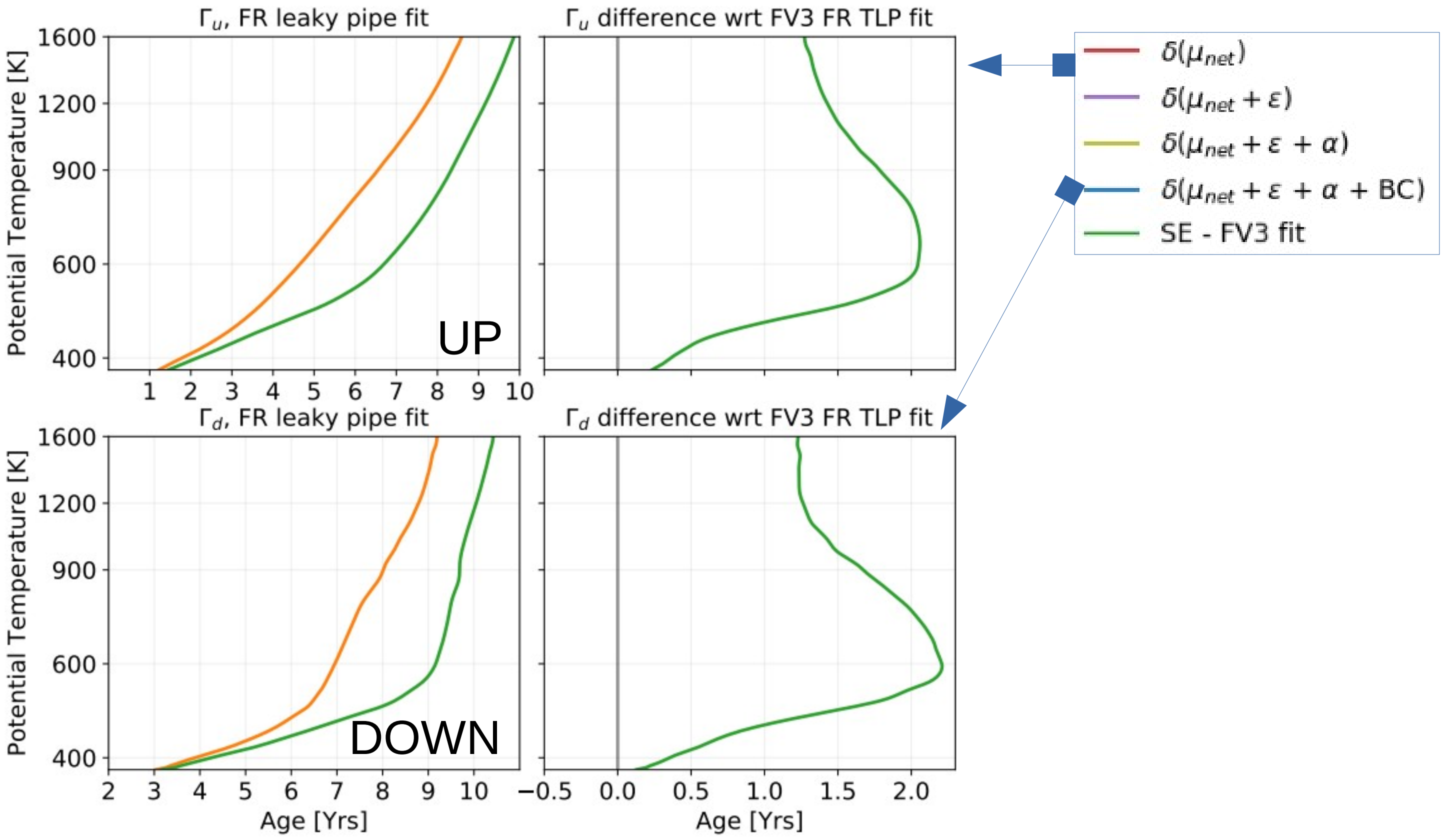
A good fit is obtained between integrated age from models and the age from vertically varying leaky pipe formulation in both the regions.

Isolating the contribution of different factors to transport

$$\Gamma_{SE} - \Gamma_{FV3} = \delta(W_T) + \delta(\mu_{mix}) + \delta(\alpha) + \delta(\text{diffusion}) + \delta(\text{tropopause boun. cond.})$$

i.e. mixing efficiency





- We start with leaky pipe fit of FV3 climate model (in orange) and *incrementally* force the leaky pipe model with the parameters of SE model (in green).

Isolating the contribution of different factors to transport

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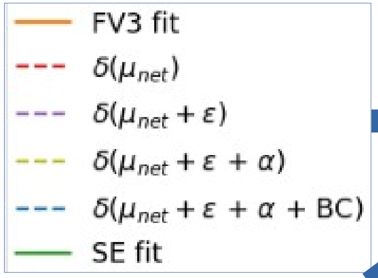

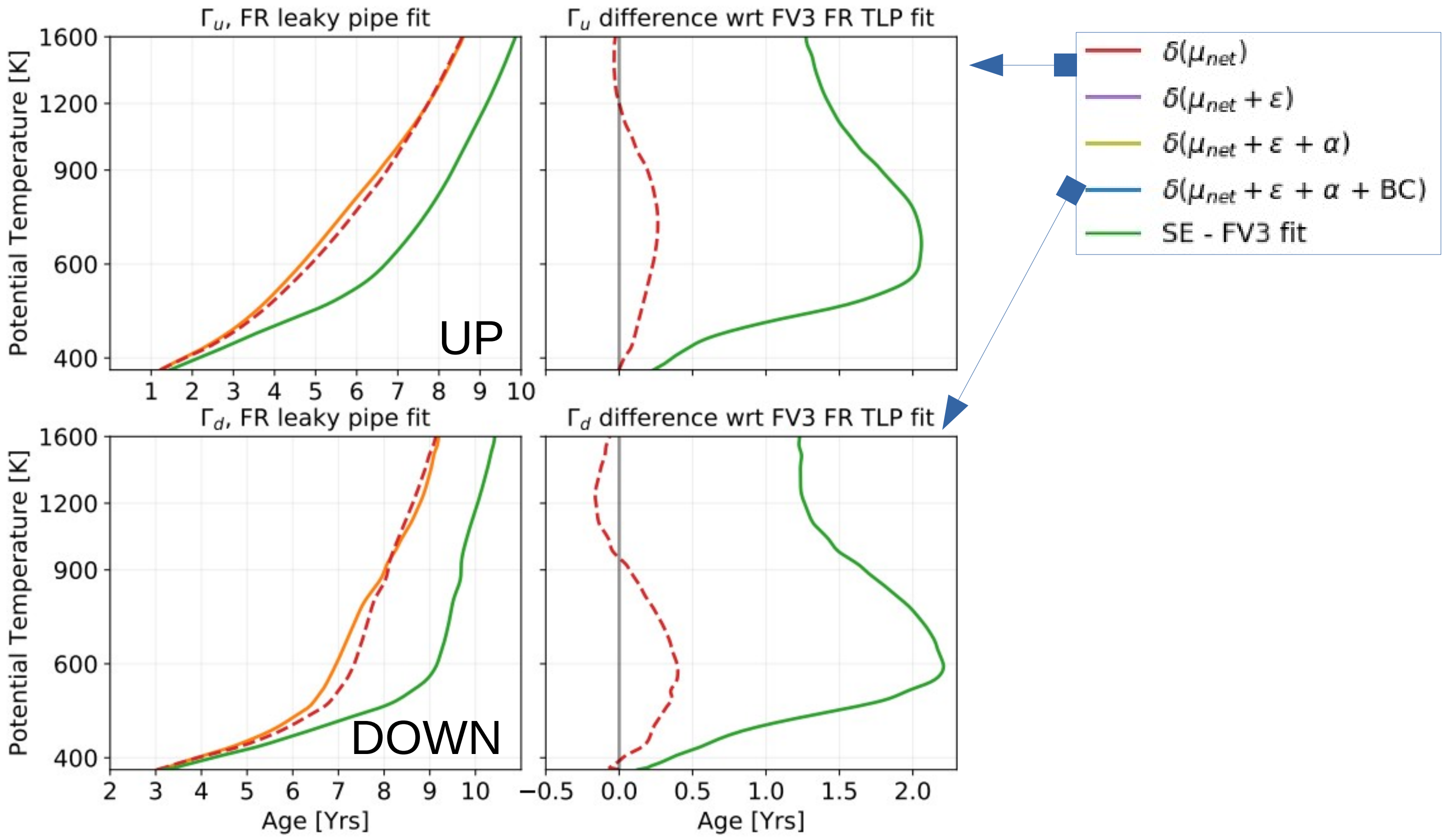

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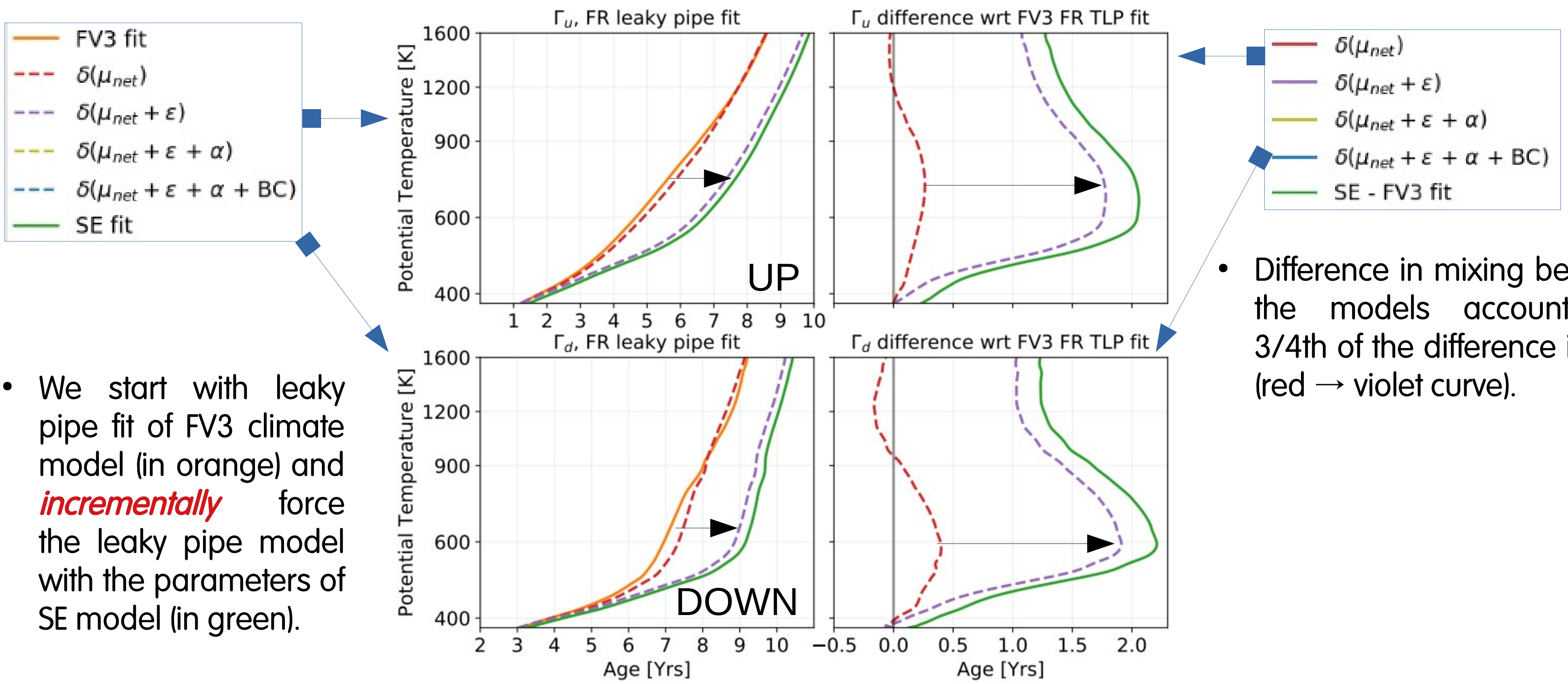

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- Difference in mixing between the models accounts for 3/4th of the difference in age (red → violet curve).

Isolating the contribution of different factors to transport

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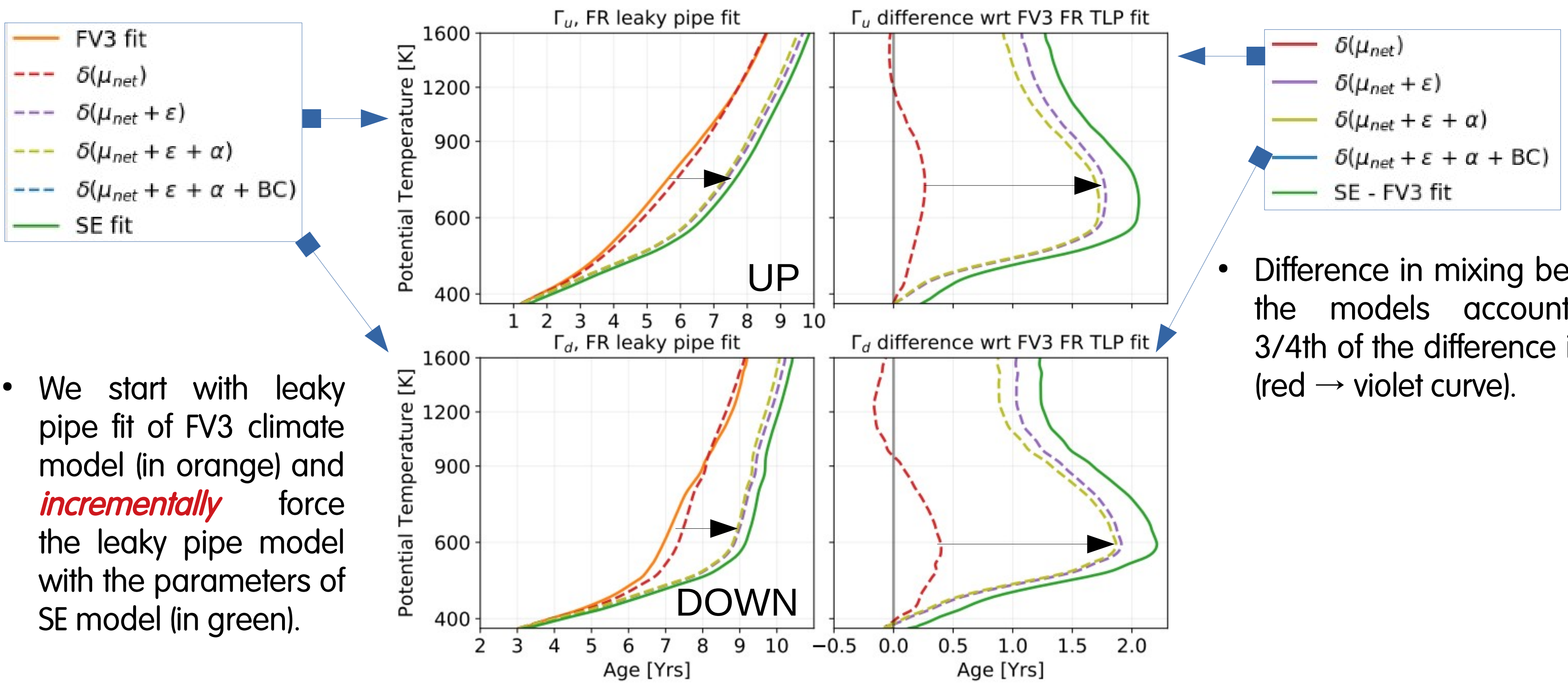

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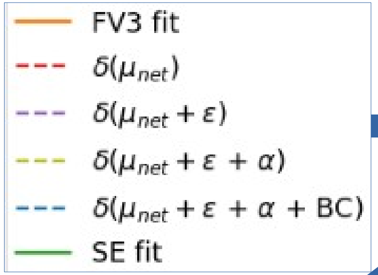

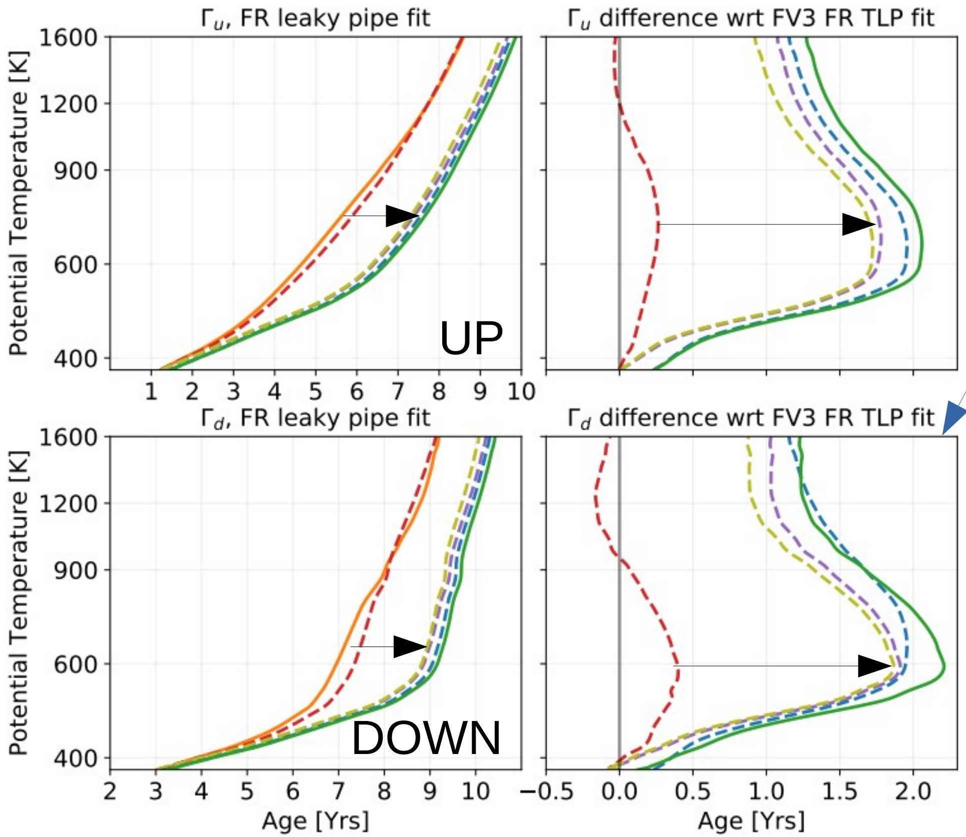

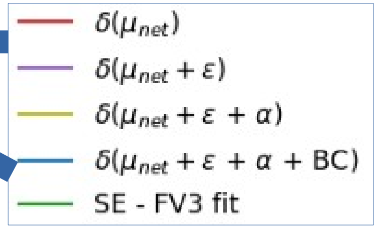

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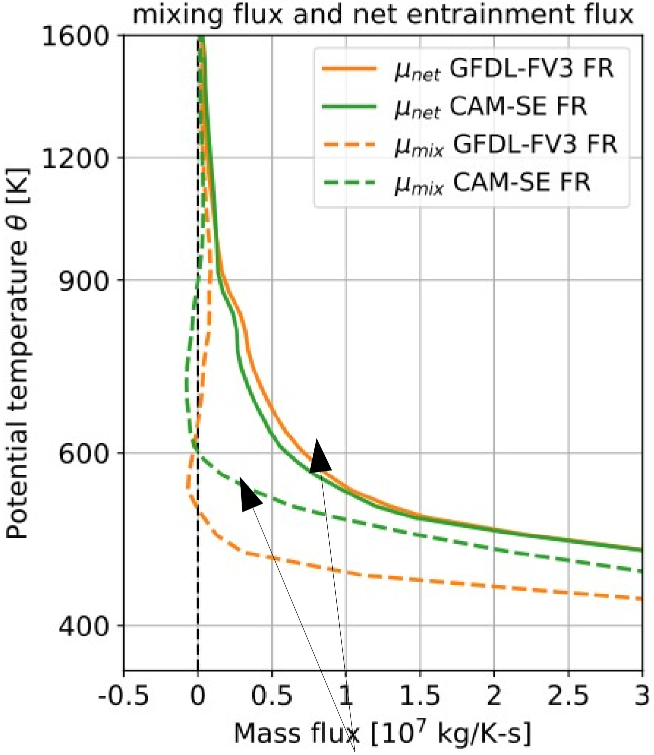
- Difference in mixing between the models accounts for 3/4th of the difference in age (red → violet curve).

- The residual difference (between solid green and dashed blue) **represents differences due to numerical diffusion**

The extratropical-tropical mixing fluxes are different indeed!

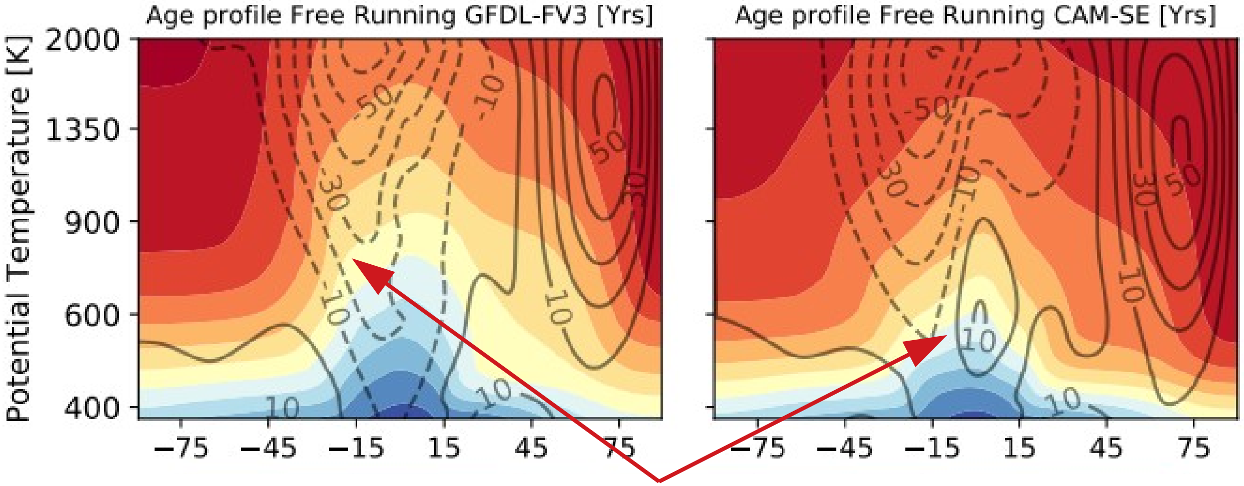
$$\frac{\partial \Gamma_u}{\partial \theta} = \frac{\sigma}{\mathcal{M}} + \frac{\mu_{mix} \Delta\Gamma}{\mathcal{M}}$$



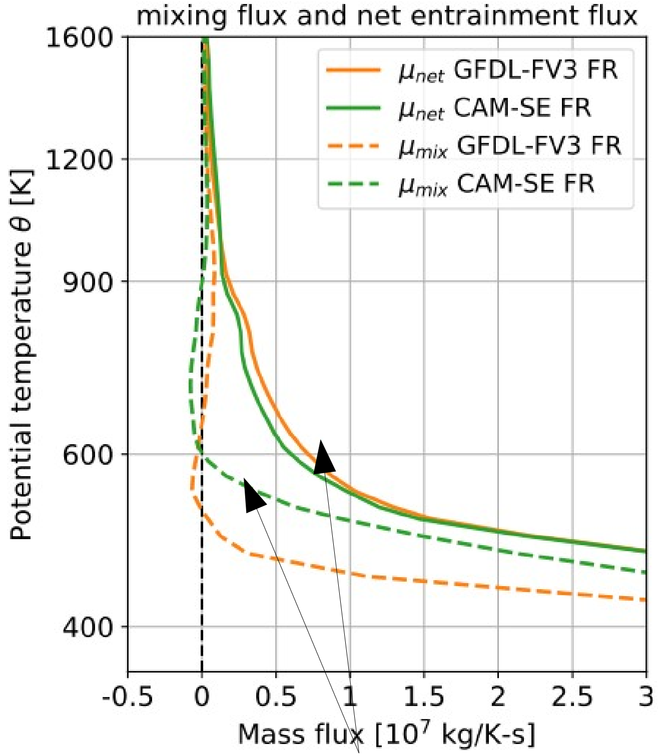
Higher mixing flux in SE as compared to FV3

The extratropical-tropical mixing fluxes are different indeed!

$$\frac{\partial \Gamma_u}{\partial \theta} = \frac{\sigma}{\mathcal{M}} + \frac{\mu_{mix} \Delta\Gamma}{\mathcal{M}}$$



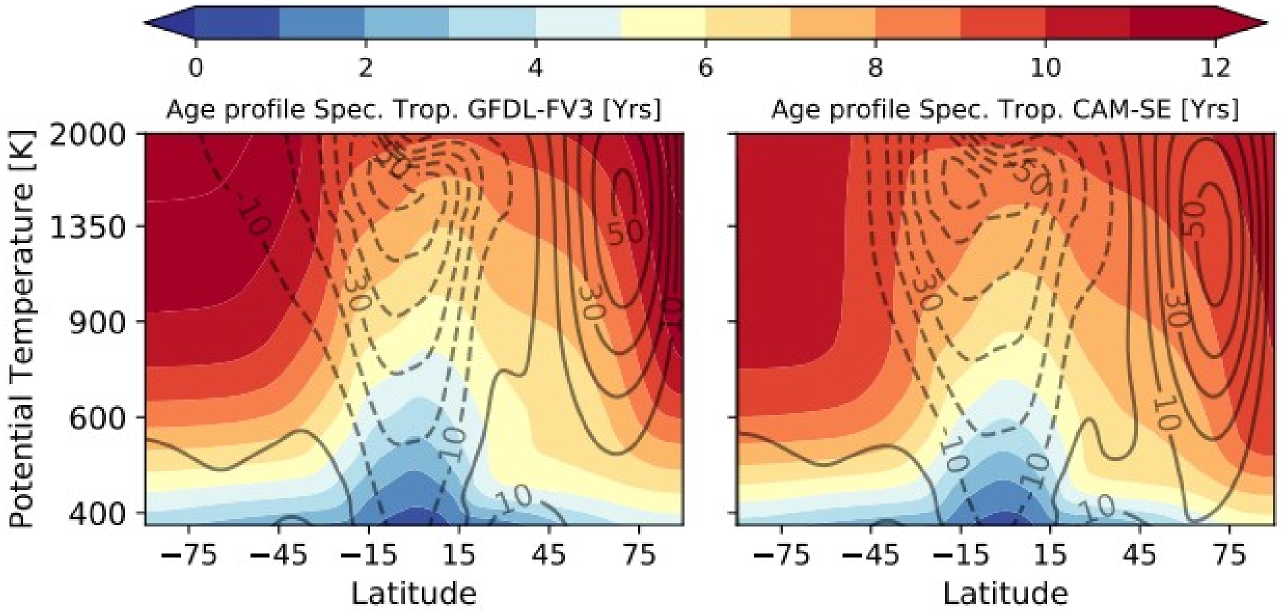
Very different tropical climatology among the two models



Higher mixing flux in SE as compared to FV3

- The tropical winds in the two models have different phases. Akin to different phases of the QBO.
- Westerlies induce more mixing between the two regions by allowing the midlatitude mixing fluxes deeper into the tropics

Does constraining the tropical winds resolve the issue?



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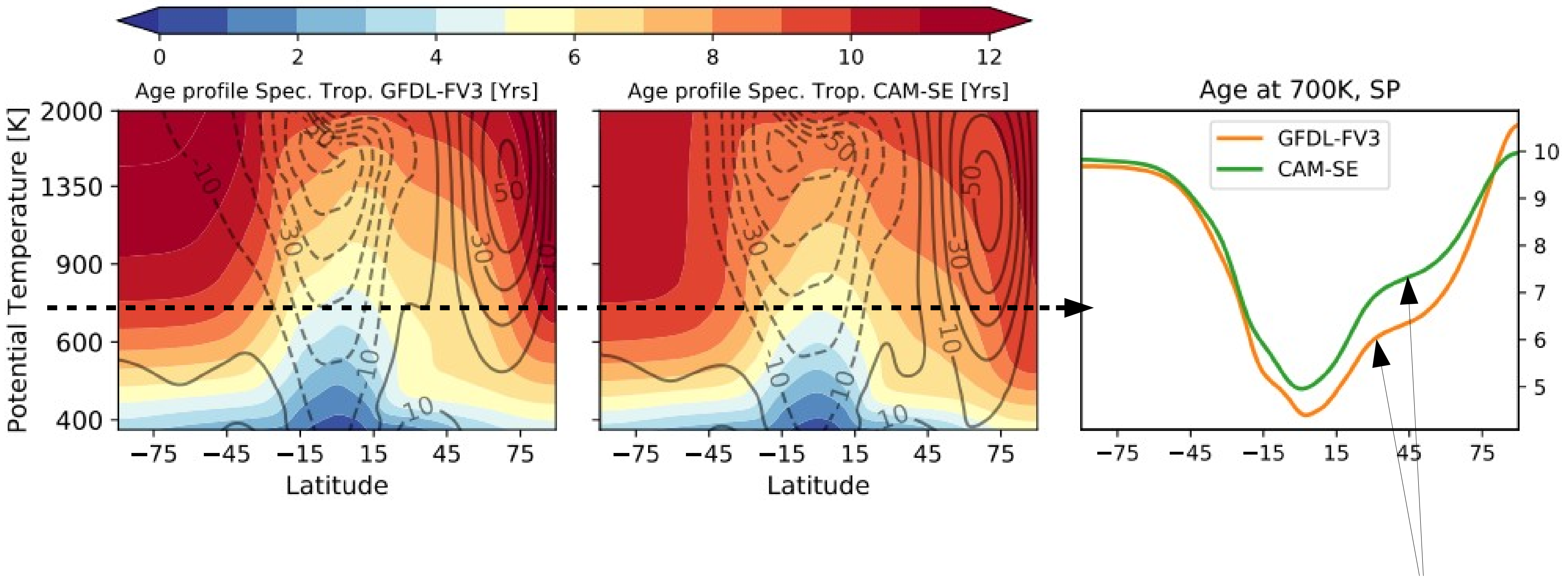

Constraining tropical winds to be identical among models drastically **reduces the age difference**. Some difference still remains.

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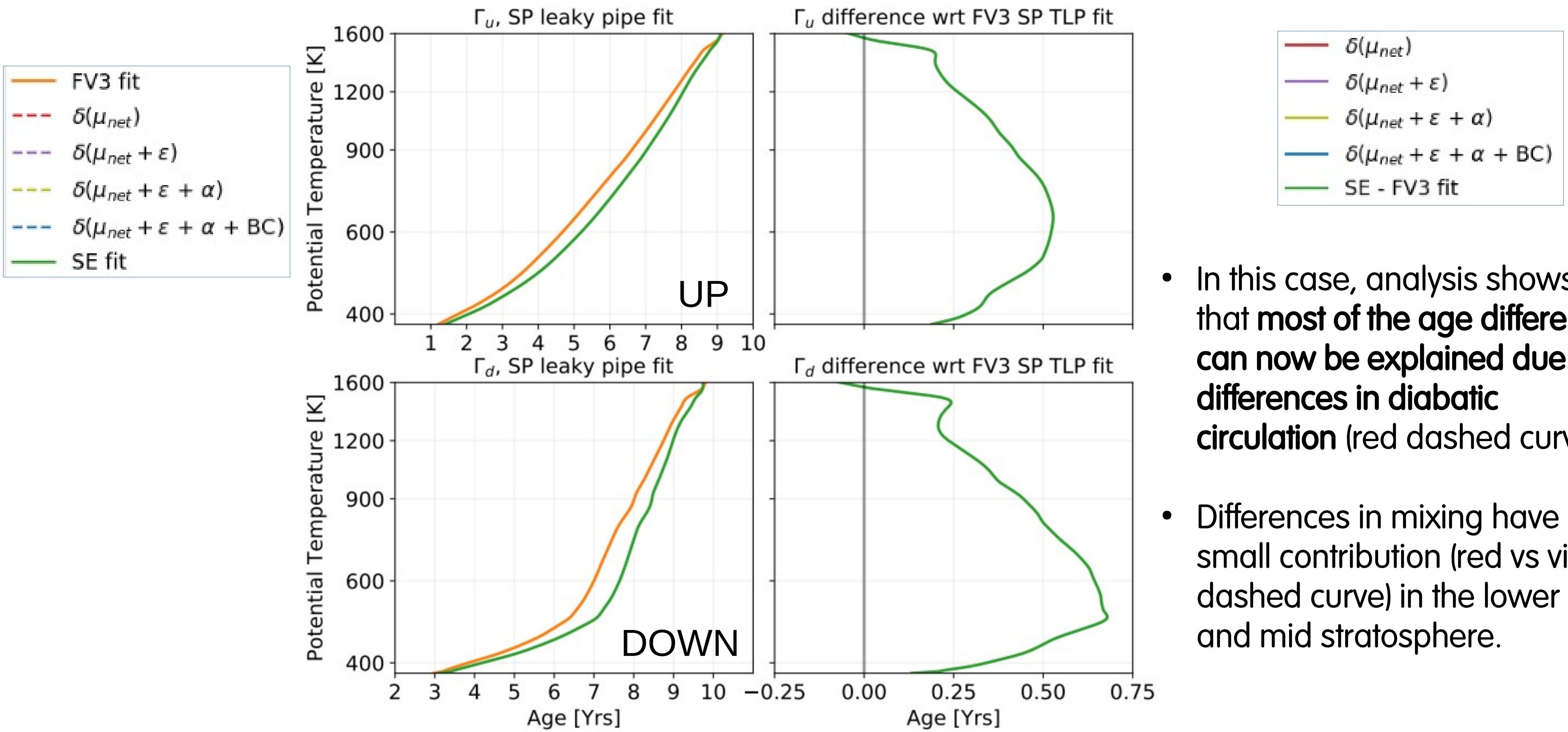

- In this case, analysis shows that **most of the age difference can now be explained due to differences in diabatic circulation** (red dashed curve).
- Differences in mixing have small contribution (red vs violet dashed curve) in the lower and mid stratosphere.

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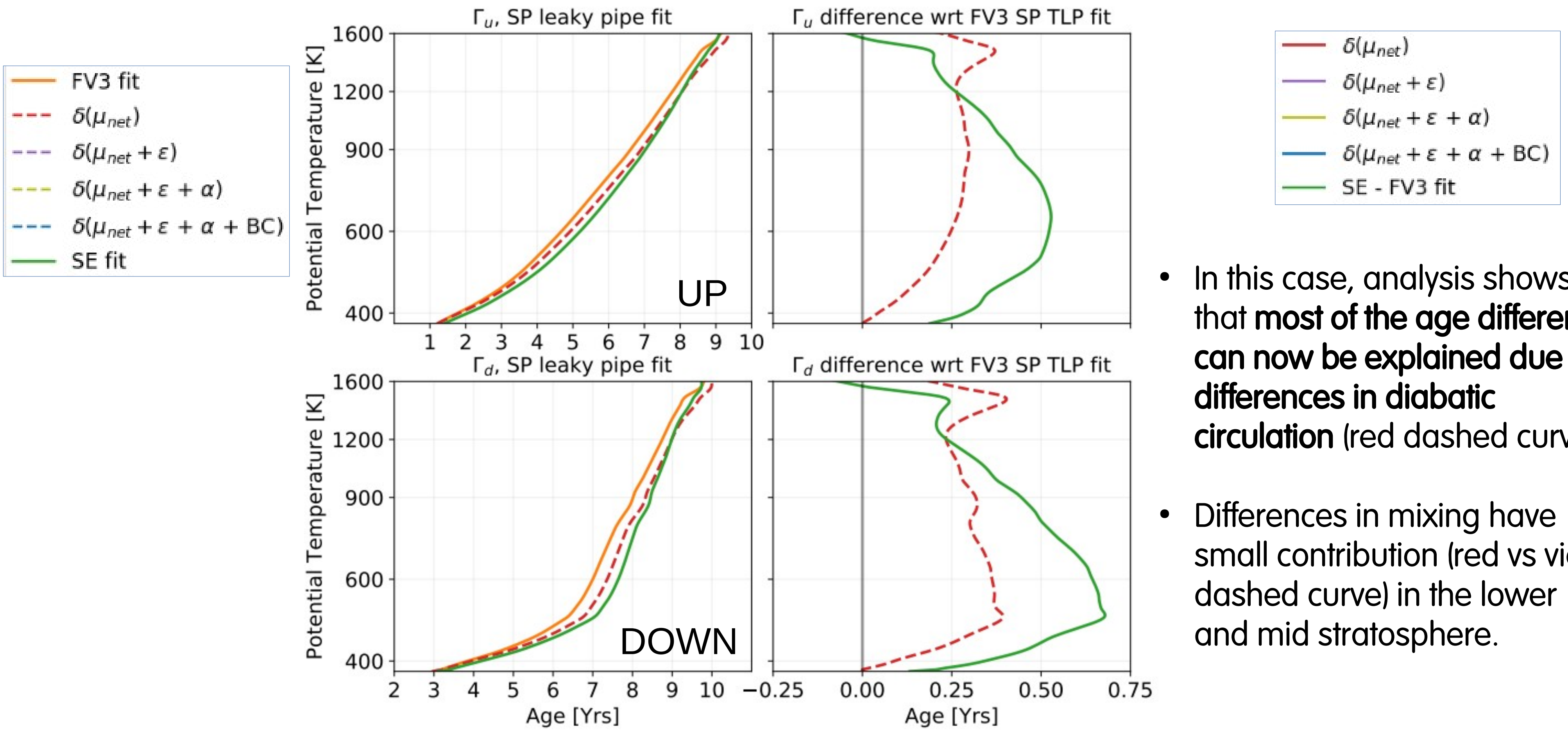

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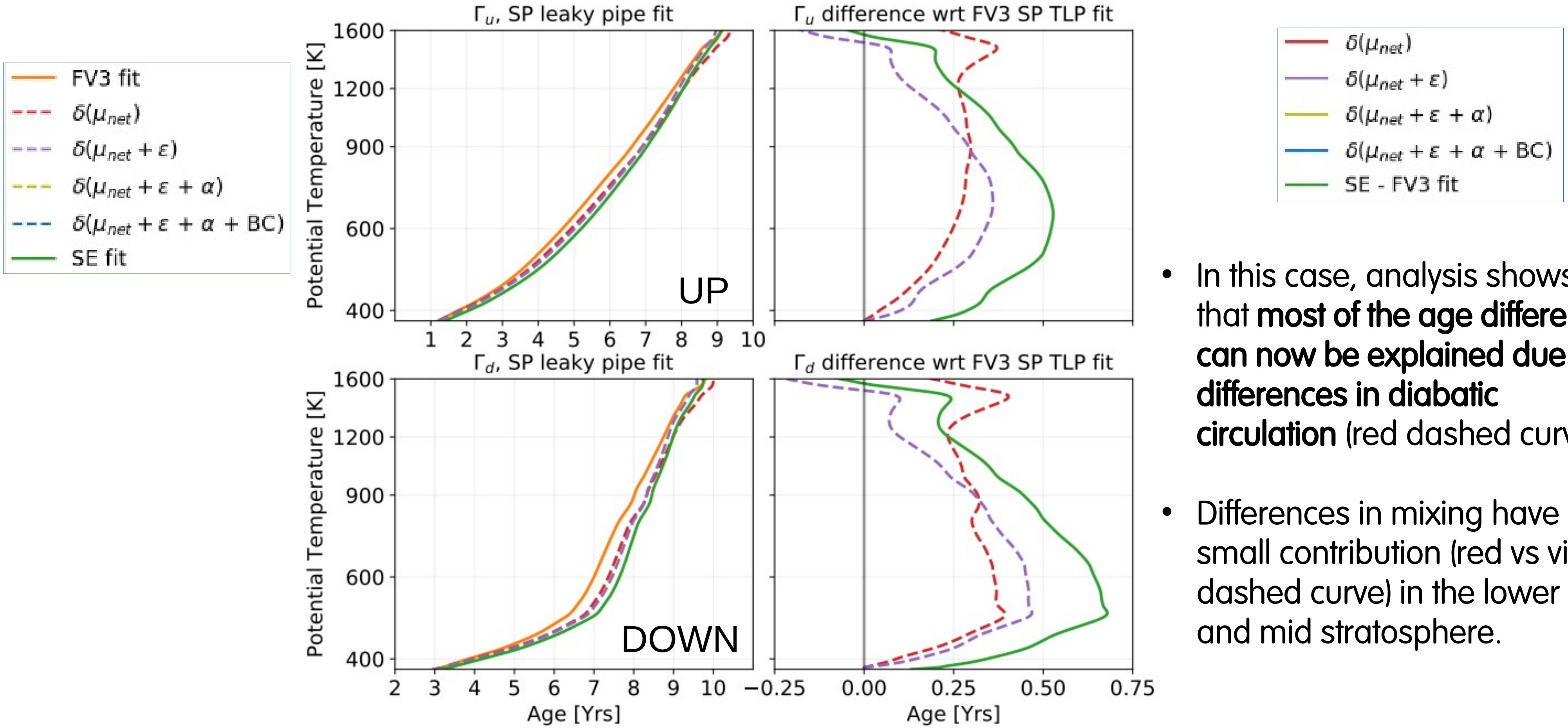

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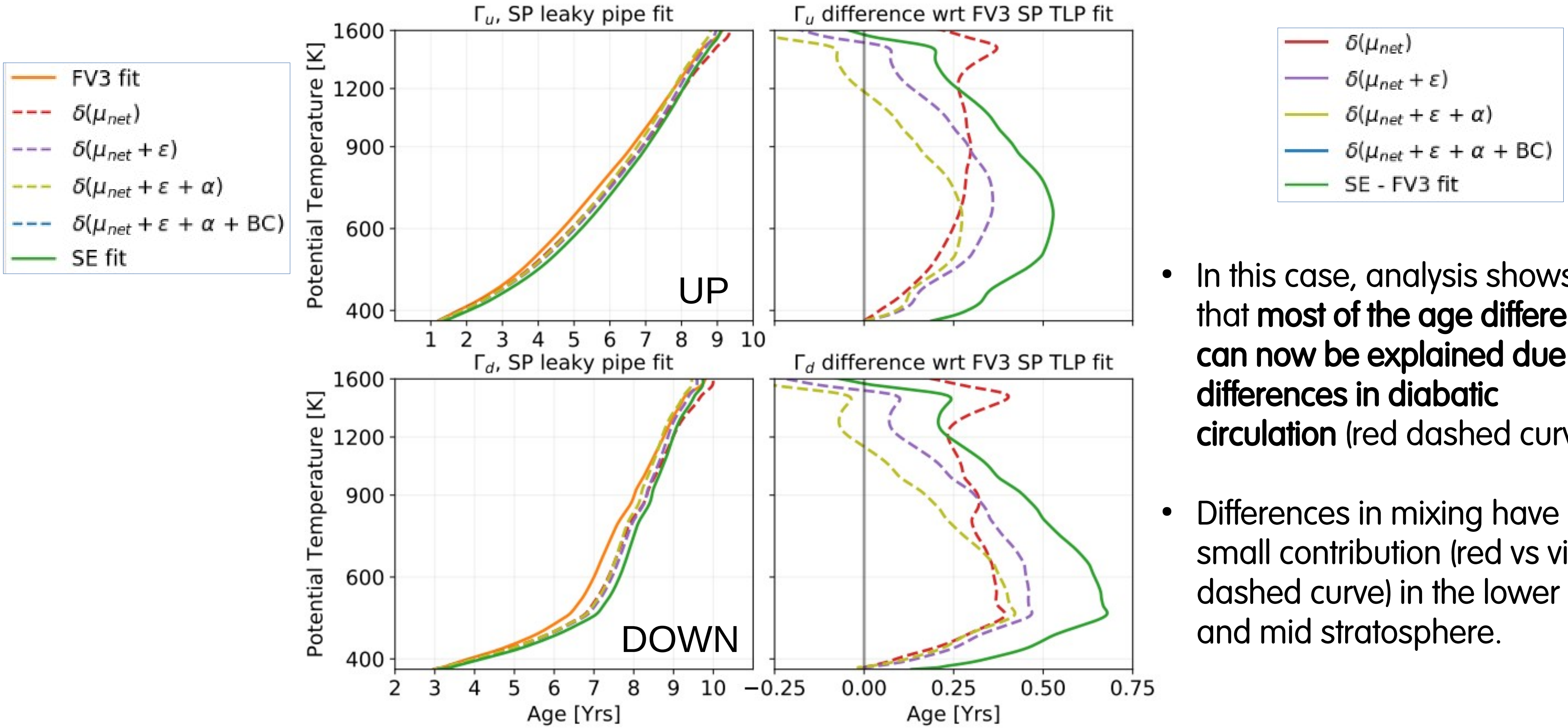

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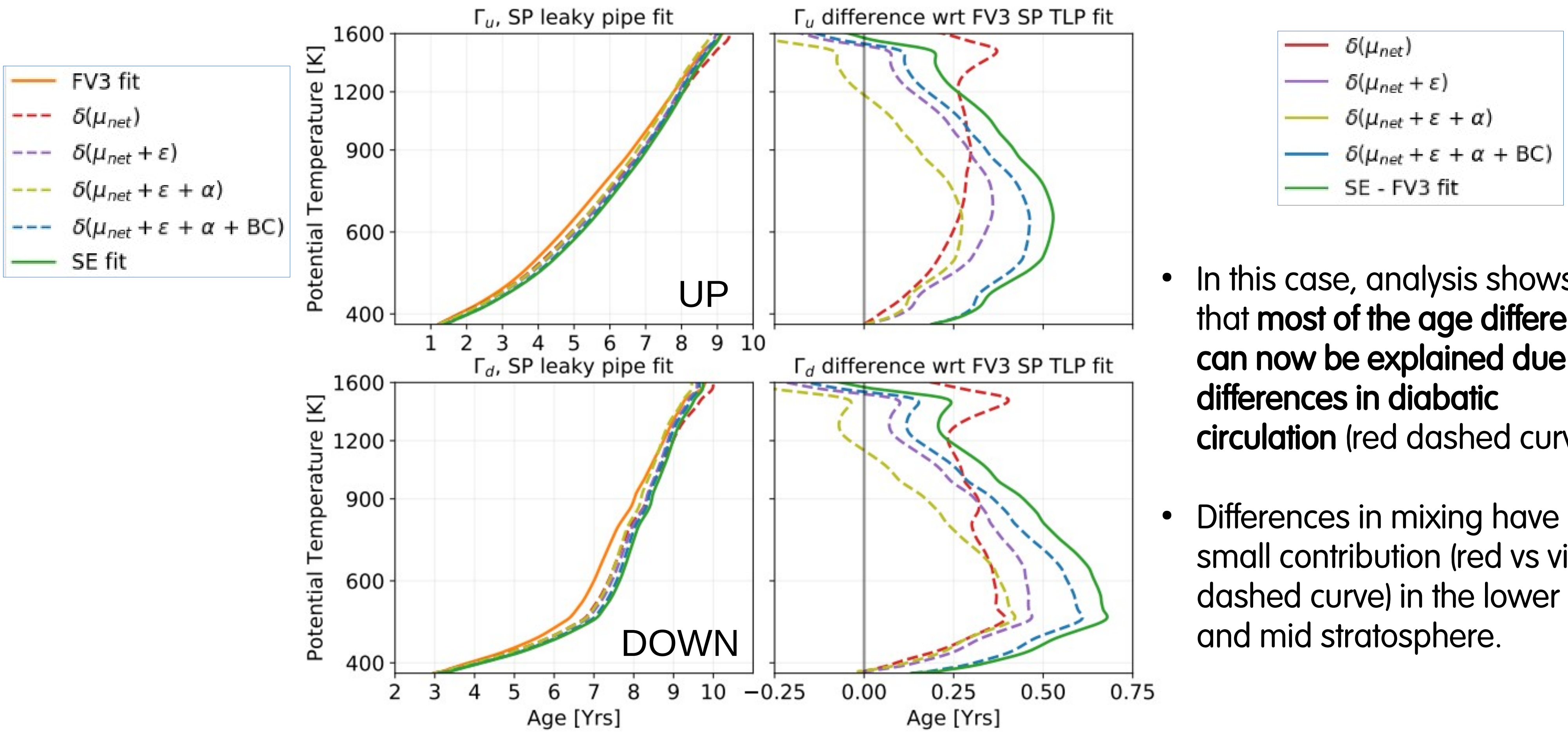

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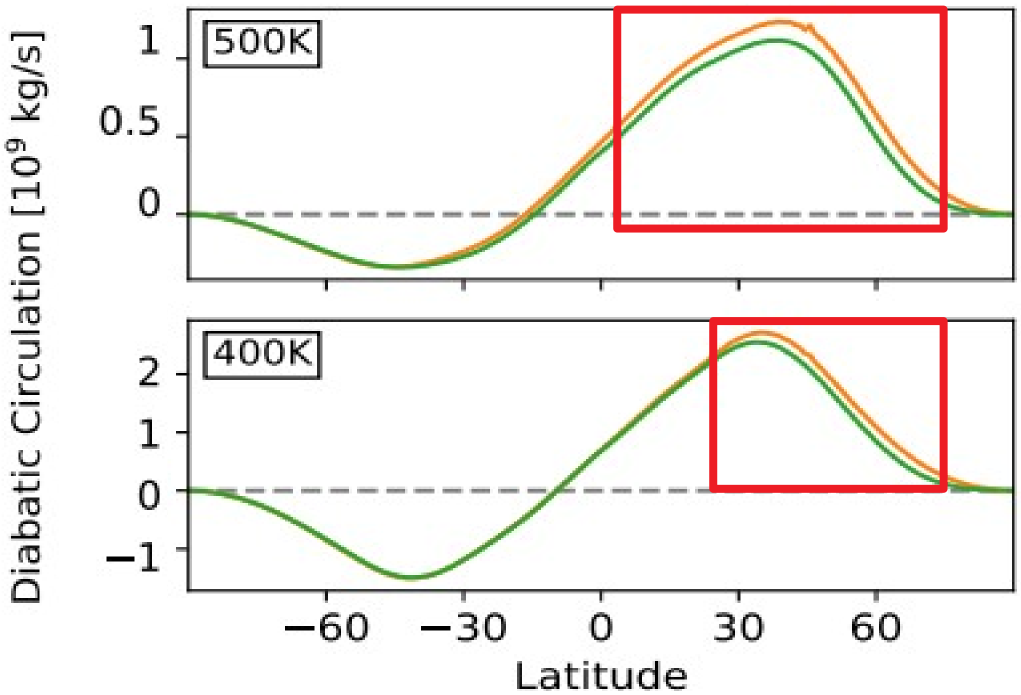
The diabatic circulation is noticeably different indeed!

$$\psi(\phi,\theta_0) = \frac{R\cos\phi}{g}\frac{1}{T}\int_0^T\int_0^{2\pi}\int_0^{p_s} vH[\theta(\lambda,\phi,p,t)-\theta_0]\,dp\,d\lambda\,dt$$

- Figure shows the diabatic mass streamfunction at two different isentropes.

- The FV3 model (in orange) develops a slightly higher diabatic circulation as compared to the SE model (in green), when the tropical winds are constrained.

- A faster circulation results in a younger age.



Conclusion

- Choice of dynamical cores is crucial while studying stratospheric dynamics and transport. State-of-the-art dynamical cores have very different stratospheric transport (age of air), despite identical forcings.
- **Following Linz et al. 2016, we link the 3D circulation to a generalized version of the theoretical leaky pipe model to analyze the transport differences.** This has been accomplished my dimensional reduction of model age and by allowing vertical variations in the original leaky pipe formulation
- Approximately 3/4th of the differences in age between the two cores was due to differences in mixing between the tropics and the extratropics, associated with differences in tropical winds. Other factors, such as diabatic circulation strength, numerical diffusion and mass distribution account for the rest of the differences.
- Then the tropical winds are constrained, age becomes more similar, but still differs by about 15% due primarily to differences in circulation

References

- Gupta, Aman, Edwin P. Gerber, and Peter H. Lauritzen: **"Numerical impacts on tracer transport: A proposed intercomparison test of Atmospheric General Circulation Models"**, *Quart. J. Roy. Meteor. Soc.*, under revision.

- Gupta, Aman, Edwin P. Gerber, R. Alan Plumb, Olivier Pauluis and Peter H. Lauritzen: **"Numerical impacts on tracer transport : Understanding biases in dynamical cores with the leaky pipe framework."**, *J. Atmos. Sci.*, in prep.

- Linz, Marianna, R. Alan Plumb, Edwin P. Gerber, Douglas E. Kinnison, and Aman Gupta: **"Stratospheric adiabatic mixing rate derived from the vertical age gradient"**, in prep.